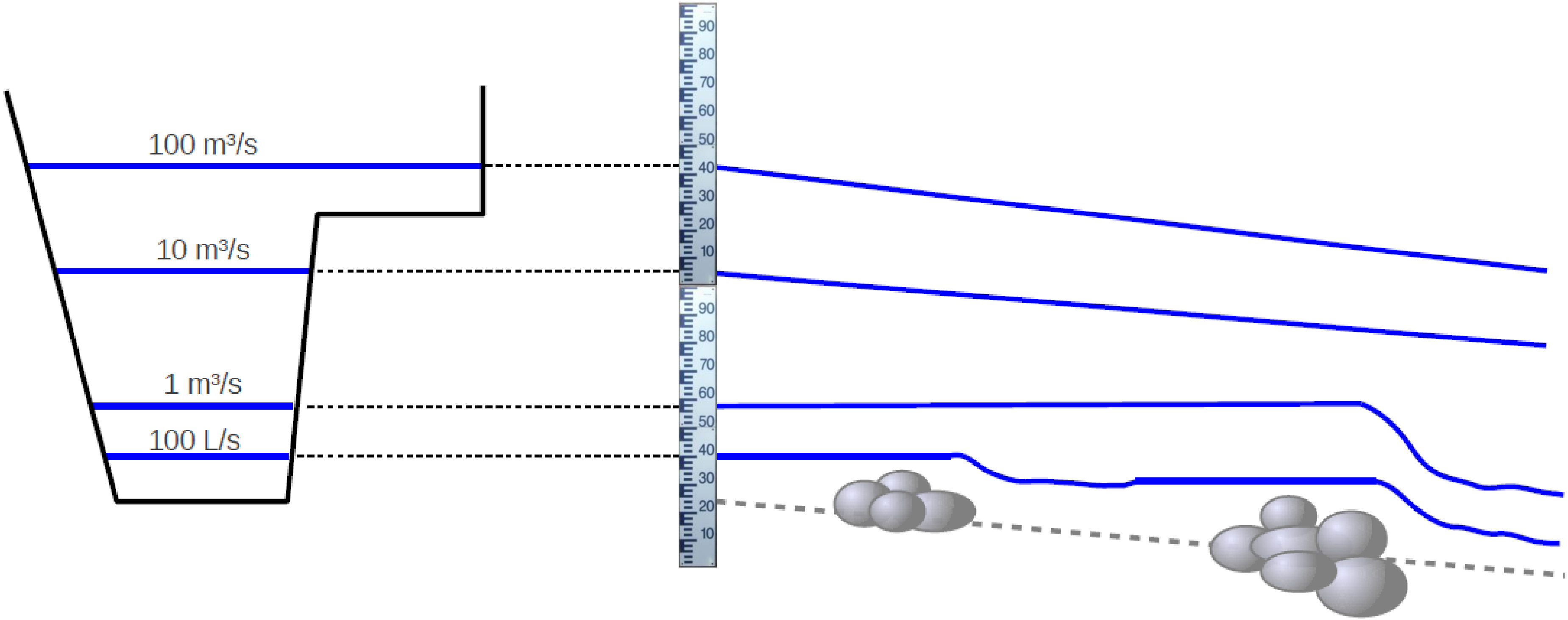
October 2022

The Bayesian estimation of rating curves: principles of the BaRatin method



Jérôme LE COZ
INRAE, UR RiverLy, Lyon, France

The BaRatin method for rating curves

- ✓ Introduction
- ✓ Hydraulic principles behind the rating curve
- ✓ Rating curve estimation
- ✓ Going further

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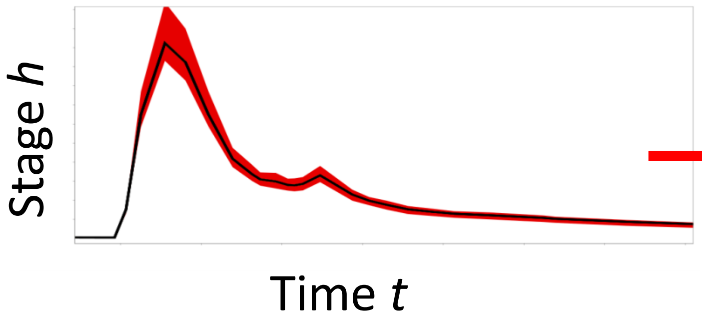
Establishing probabilistic streamflow data

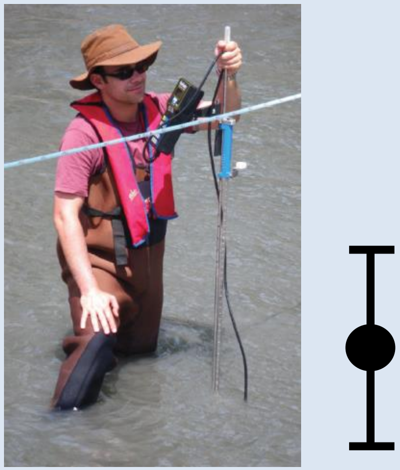


Water level series

$h(t)$

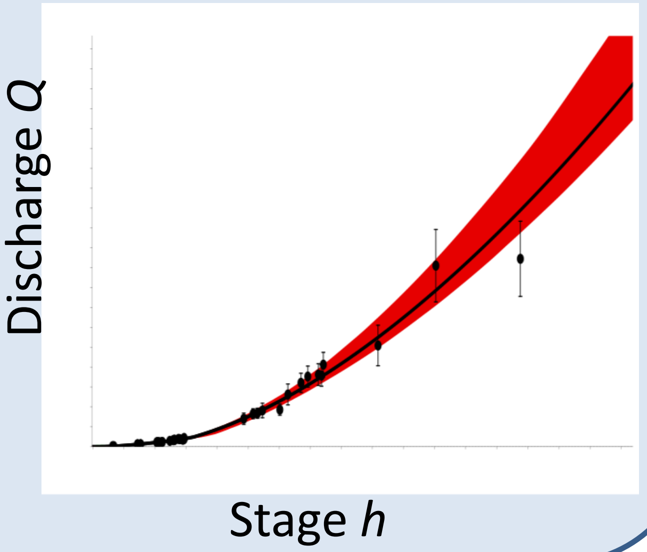
Gaugings

(Q_i, h_i)



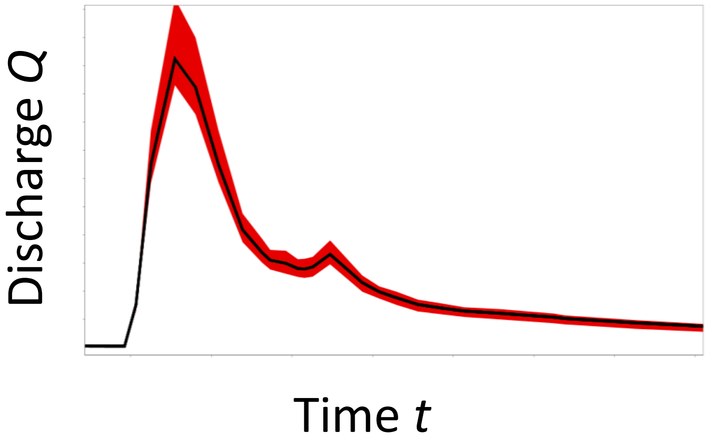
Rating curve

$Q(h)$



Streamflow series

$Q(t)$



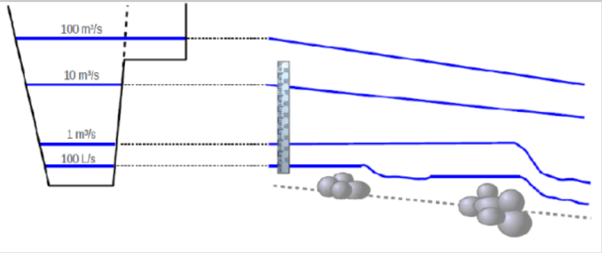
BaRatin (Bayesian rating curves)

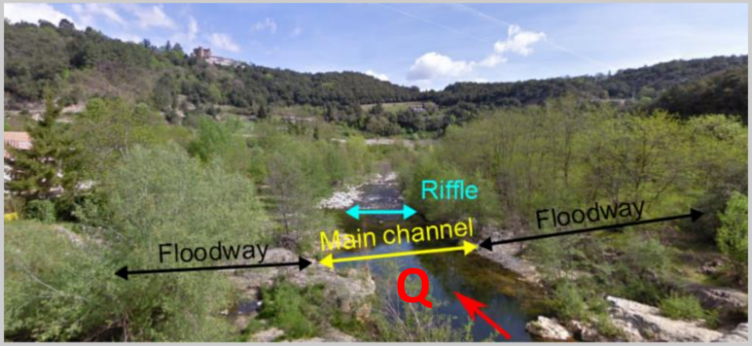
Develop a practical calibration technique to:

- Combine data (measurements) and hydraulic knowledge
- Make expert knowledge and assumptions easier to defend and review
- Account for data and hydraulics uncertainties
- Provide discharge uncertainties

DATA
MEASUREMENTS
STATISTICS

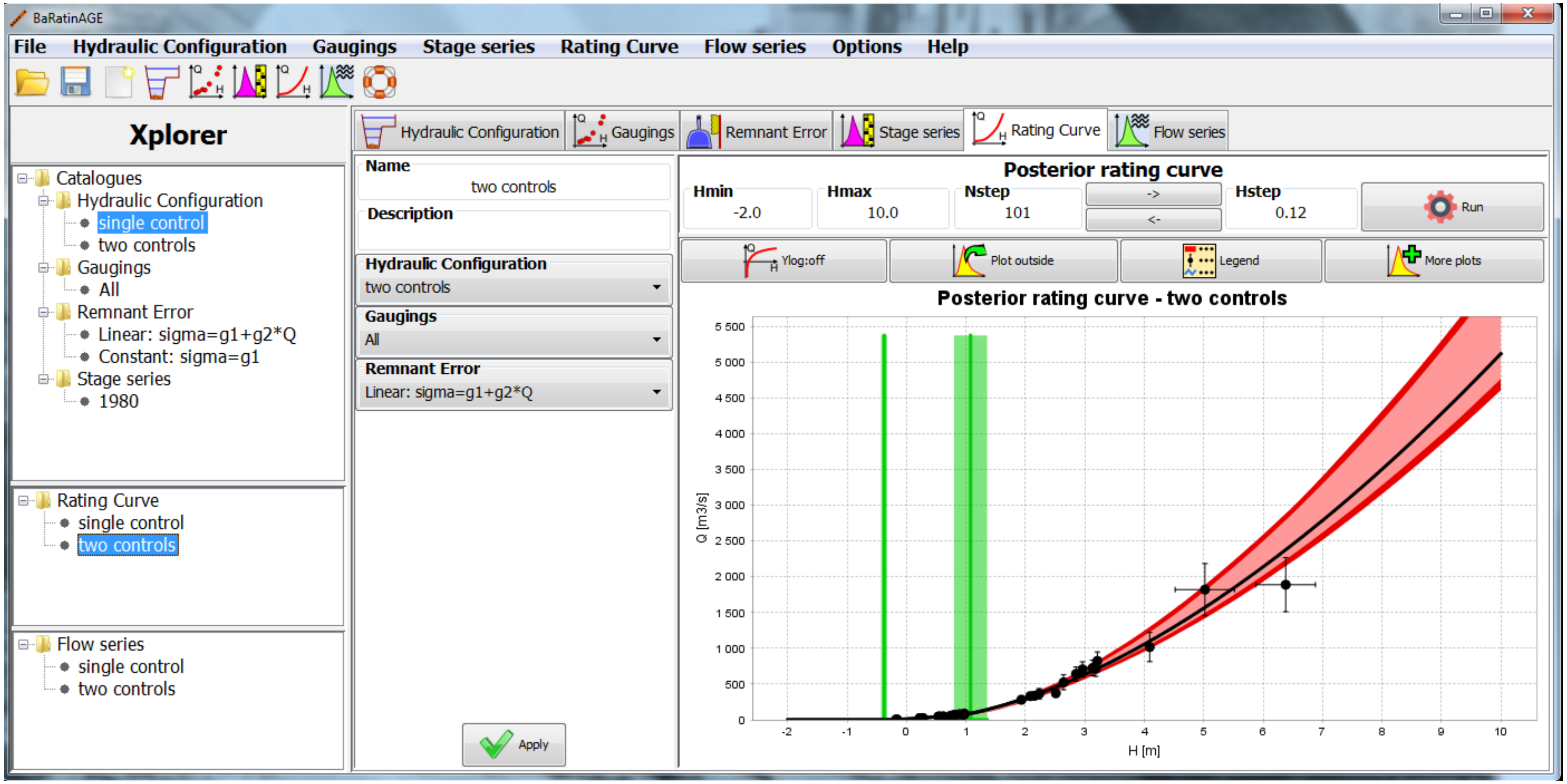
MODEL
CONTROLS
HYDRAULICS





BaRatinAGE software

- Graphical interface (Java) and user manual
- Freely available in French, English and other languages
- Open-source (GPL3), codes available on GitHub
- ~200 registered users



Catalogues
- Hydraulic Configuration
  - single control
  - two controls
- Gaugings
  - All
- Remnant Error
  - Linear: sigma=g1+g2*Q
  - Constant: sigma=g1
- Stage series
  - 1980

Rating Curve
- single control
- two controls

Flow series
- single control
- two controls

Name: two controls

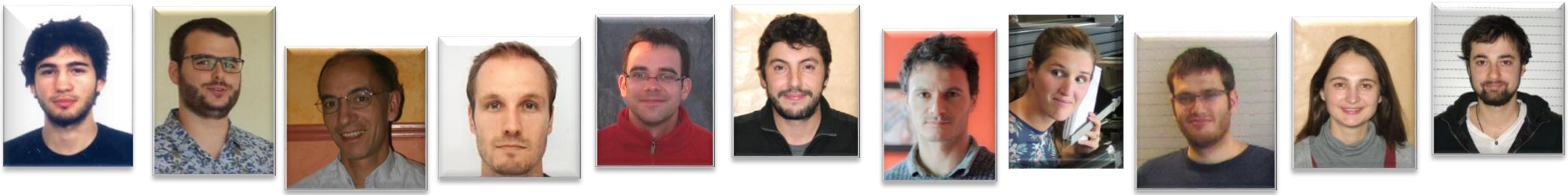
Description

Hydraulic Configuration: two controls

Gaugings: All

Remnant Error: Linear: sigma=g1+g2*Q

Hmin	Hmax	Nstep	Hstep
-2.0	10.0	101	0.12

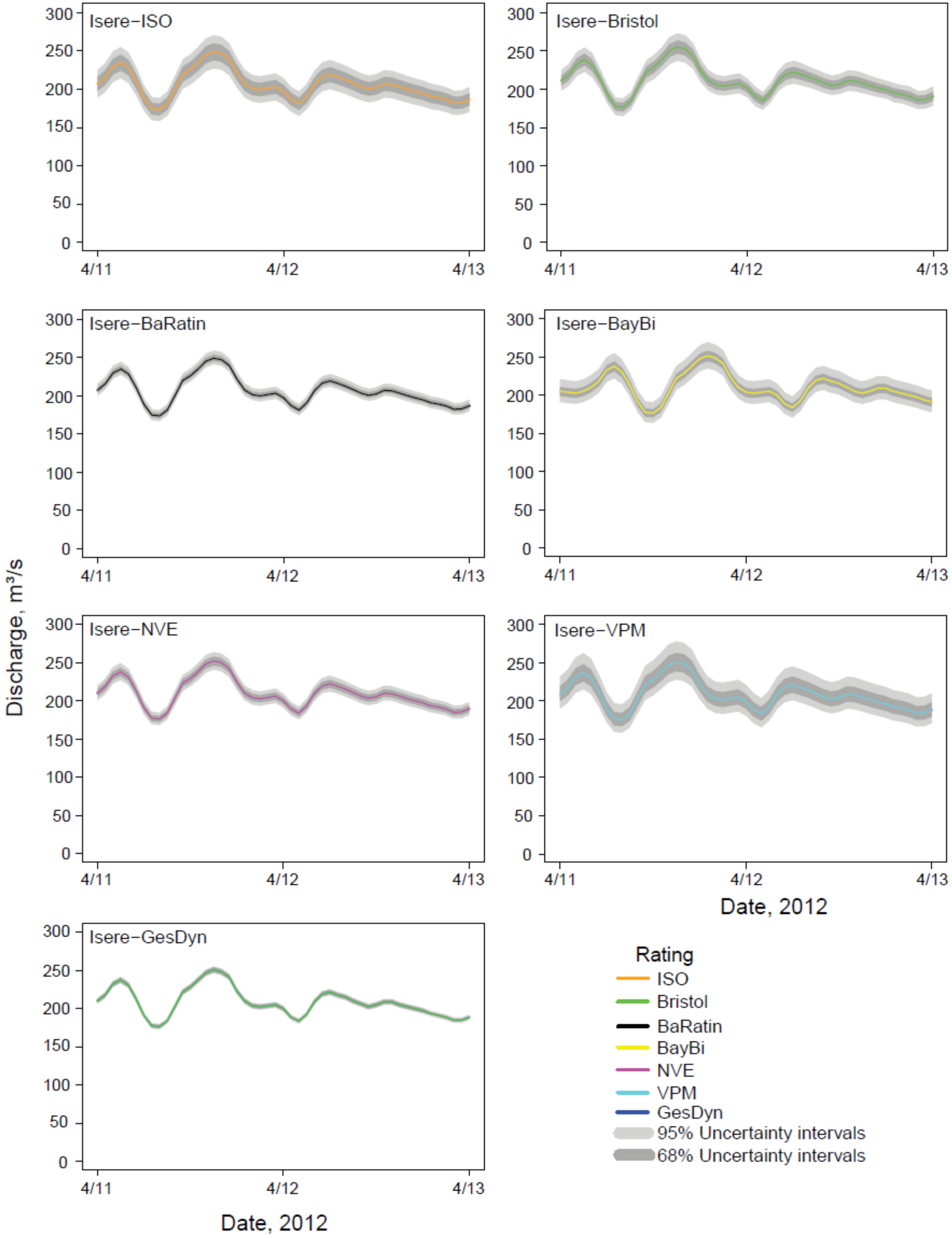


Rating curve uncertainty methods

New methods have been developed in the past decade to tackle practical and theoretical issues of the existing ISO/WMO method (which no hydrological service use routinely).

BaRatin is among the 3 that are used operationally, and likely the most widely released.

Kiang et al. (2018)
Comparison of 7 methods for rating curve uncertainty



The BaRatin method for rating curves

- ✓ Introduction
- ✓ Hydraulic principles behind the rating curve
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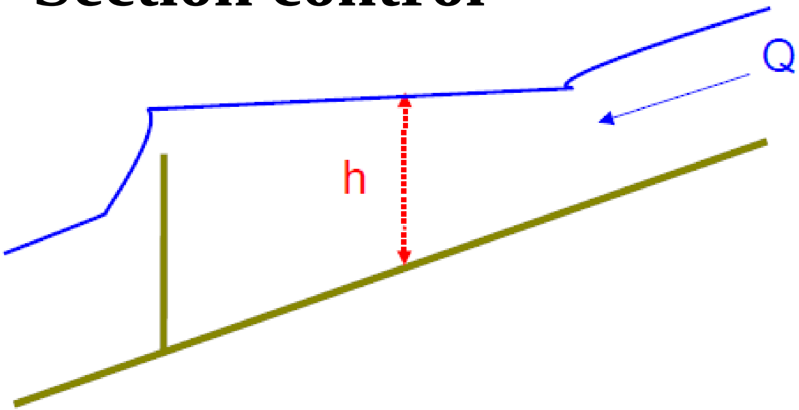
- ✓ Going further

Hydraulic controls

- Physical properties of a channel which determine the relationship between stage and discharge at a location in the channel

(World Meteorological Organization, 2012)

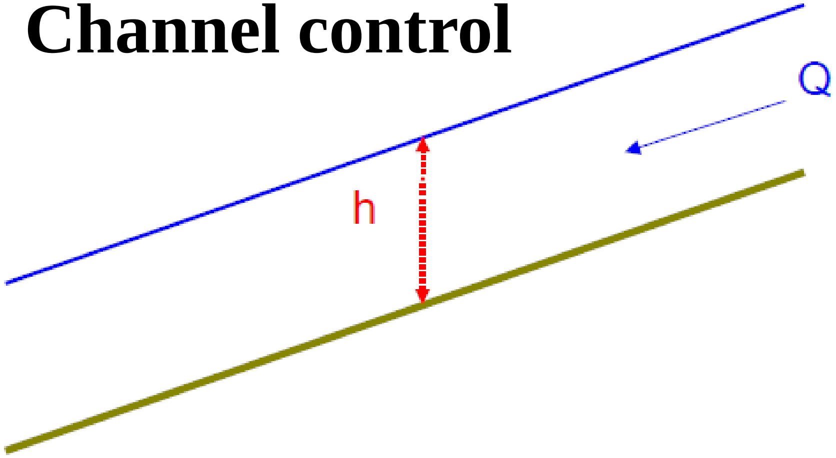
Section control



Fall (critical flow: chocked flow)

Upstream water level ~ horizontal
« Emptying bucket »

Channel control



No fall (friction-dominated flow)

Water level ~ parallel to riverbed

Section controls


Mahurangi, New Zealand


Le Golo, France


Le Gapeau, France


L'Ardèche, France

Channel controls

L’Isère, France

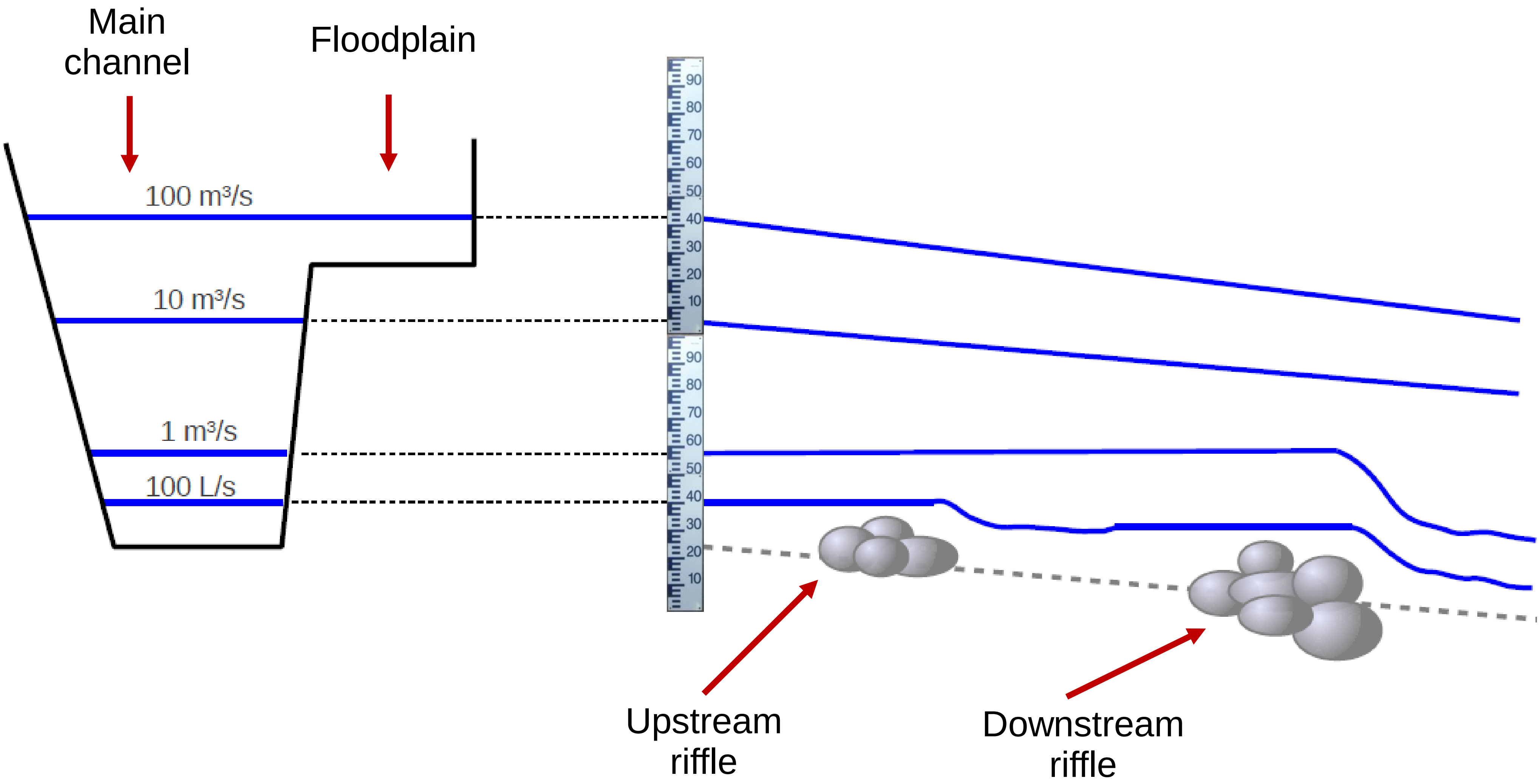
La Sarre, France

L’Ardèche, France

La Saône, France

Hydraulic controls

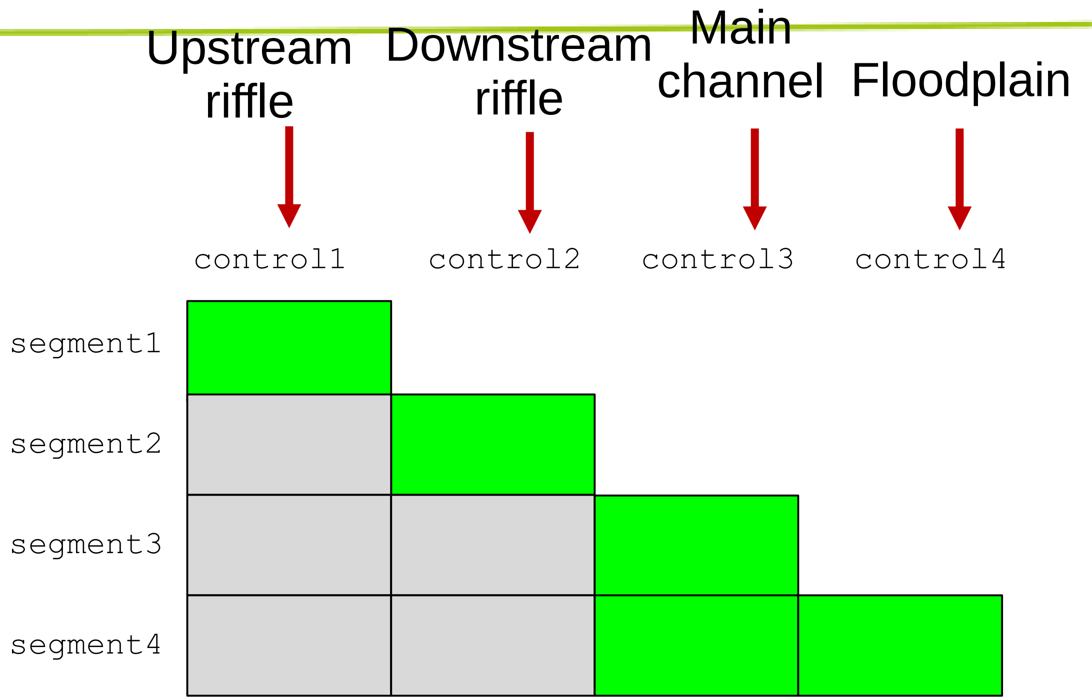
Depending on the water level, differing controls may appear or disappear. Several controls may add up.



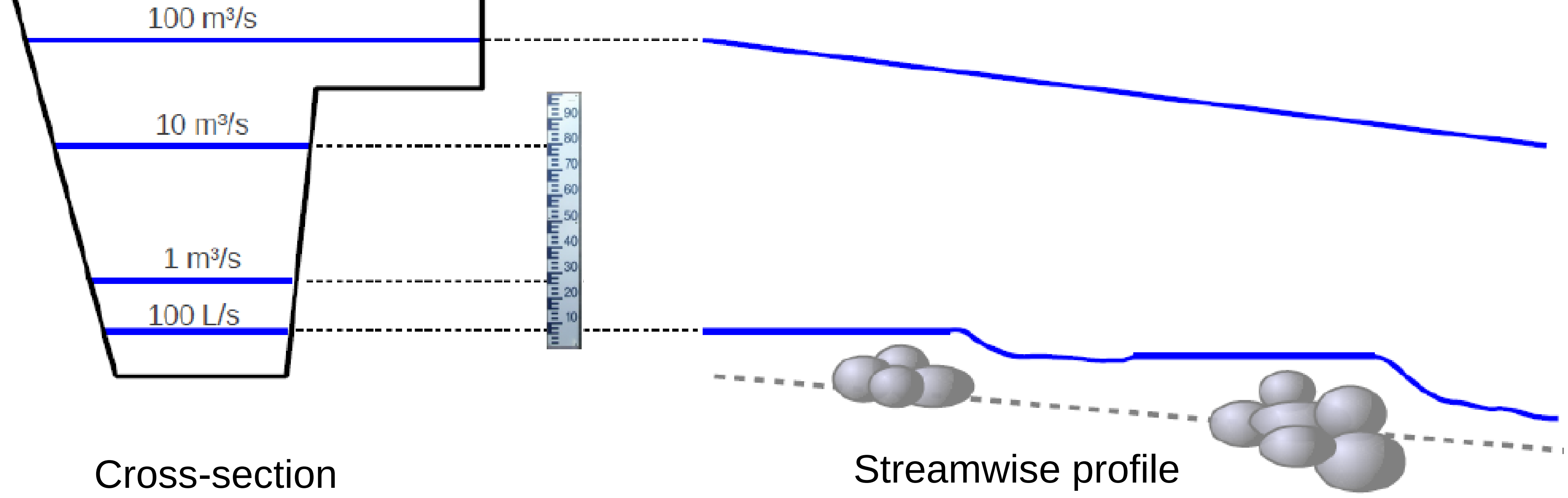
Hydraulic analysis

The main controls are identified or assumed.

The succession of controls over rating curve segments is represented by a matrix.

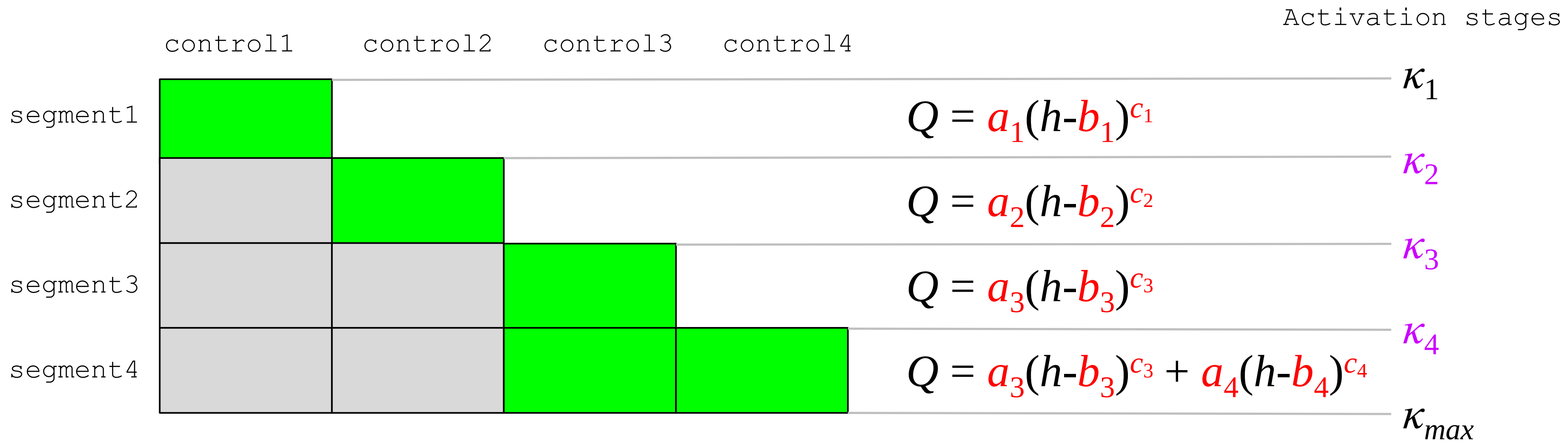


	control1	control2	control3	control4
segment1	🟩			
segment2	⬜	🟩		
segment3	⬜	⬜	🟩	
segment4	⬜	⬜	🟩	🟩



Rating curve equation

Each control can be modelled as: $Q = a(h-b)^c$



Activation stages

	control1	control2	control3	control4
segment1	🟩			
segment2	▨	🟩		
segment3	▨	▨	🟩	
segment4	▨	▨	🟩	🟩

κ_1

$Q = a_1(h-b_1)^{c_1}$

κ_2

$Q = a_2(h-b_2)^{c_2}$

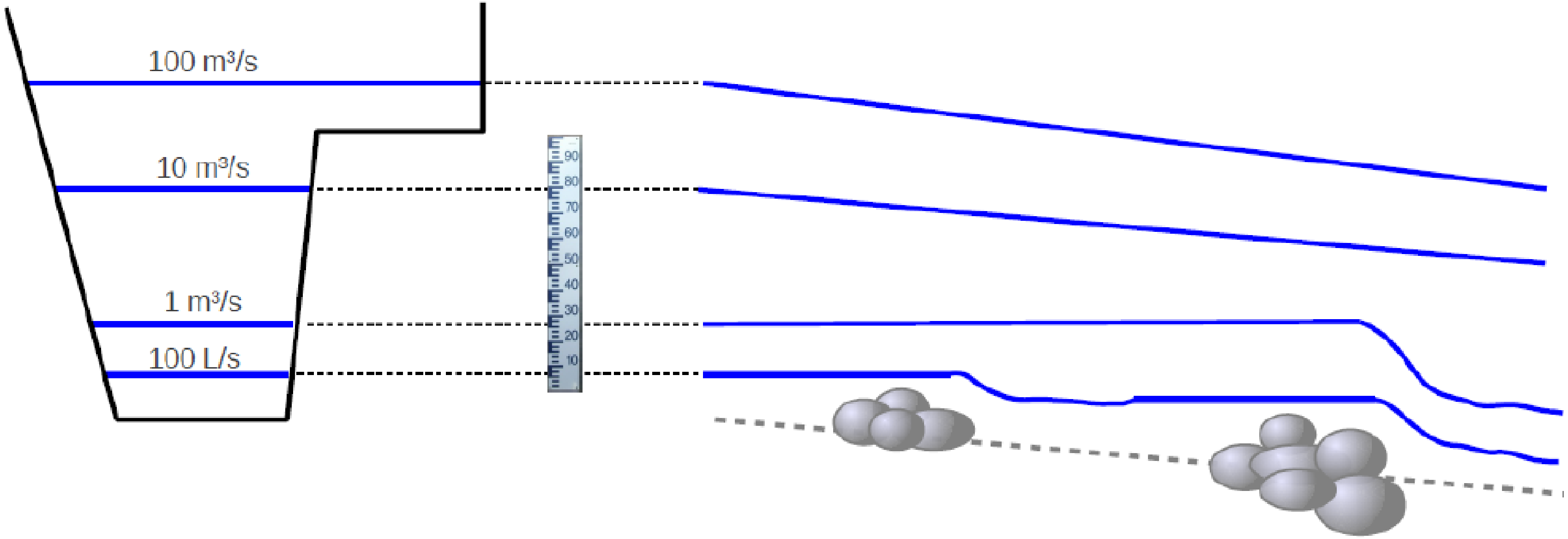
κ_3

$Q = a_3(h-b_3)^{c_3}$

κ_4

$Q = a_3(h-b_3)^{c_3} + a_4(h-b_4)^{c_4}$

κ_{max}



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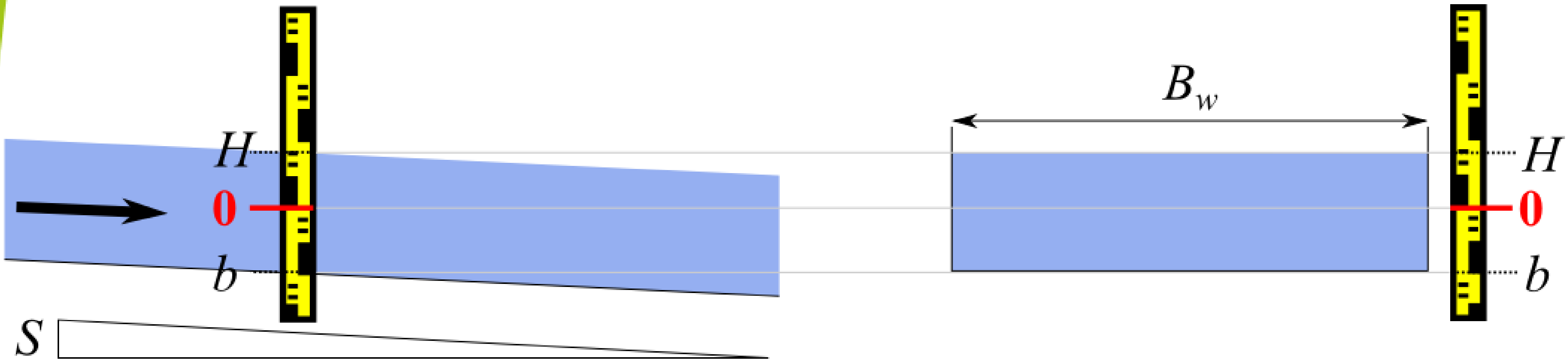
Standard controls in BaRatinAGE

- Wide, rectangular channel (fairly uniform flow)

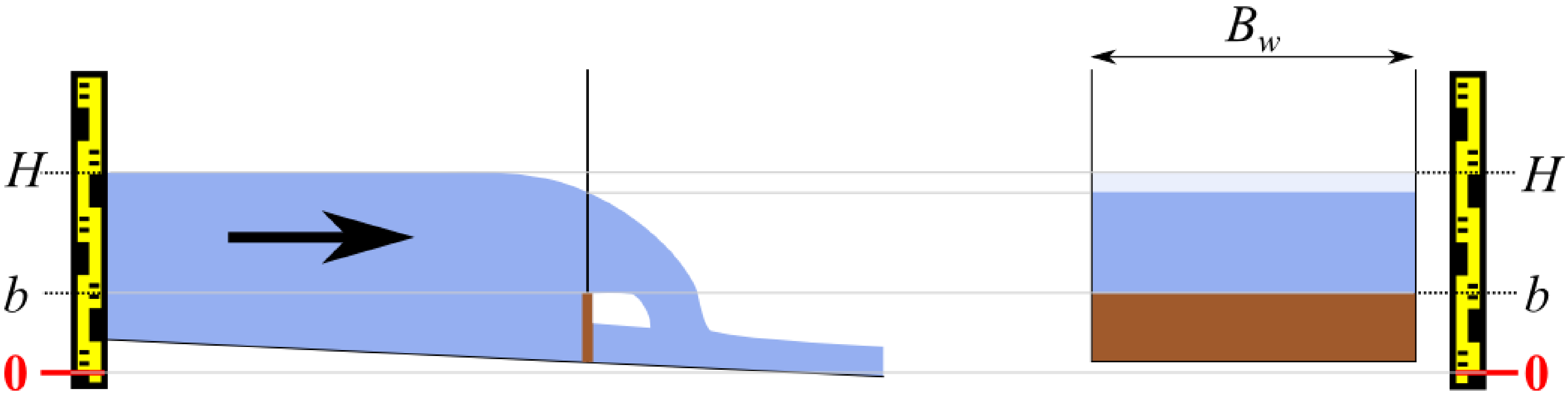


$$Q(H) = \underbrace{K_S\sqrt{S}\,B_w}_{a}\,(H-b)^{5/3}$$

Strickler coefficient

Standard controls in BaRatinAGE

- Rectangular weir / natural riffle



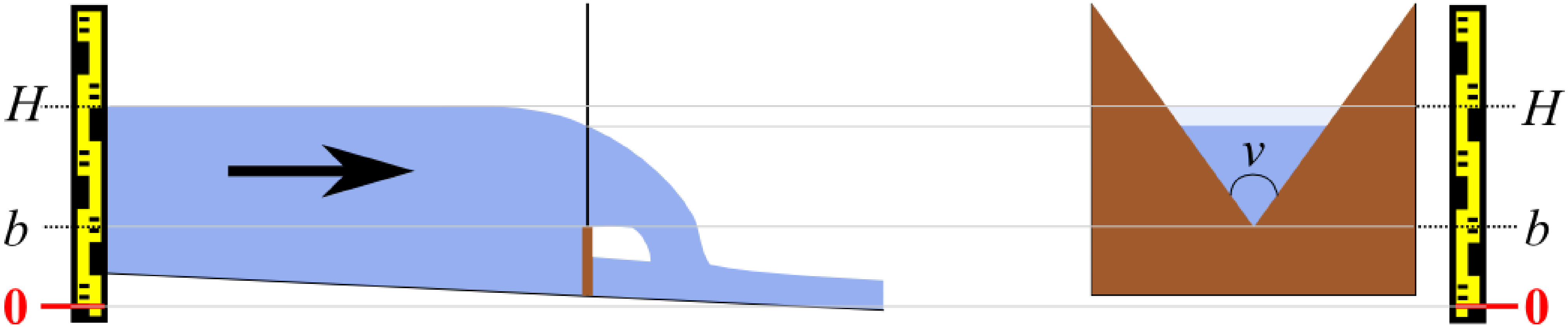
$$Q(H) = \underbrace{C_r\sqrt{2g}\,B_w}_{a}(H-b)^{1.5}$$

Discharge coefficient ≈ 0.4

gravity ≈ 9.81 m/s²

Standard controls in BaRatinAGE

- Triangular weir / V-notch

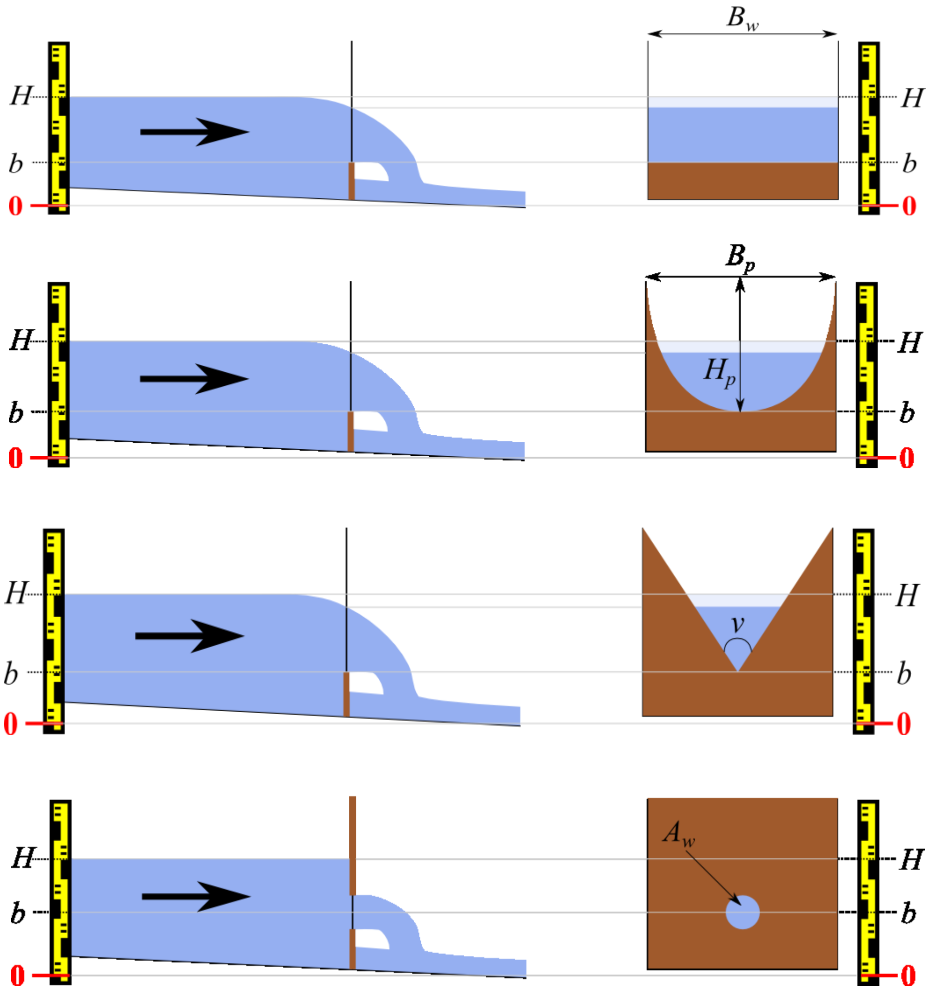


$$Q(H) = \underbrace{C_t\sqrt{2g}\ \tan(v/2)}_{a}\,(H-b)^{2.5}$$

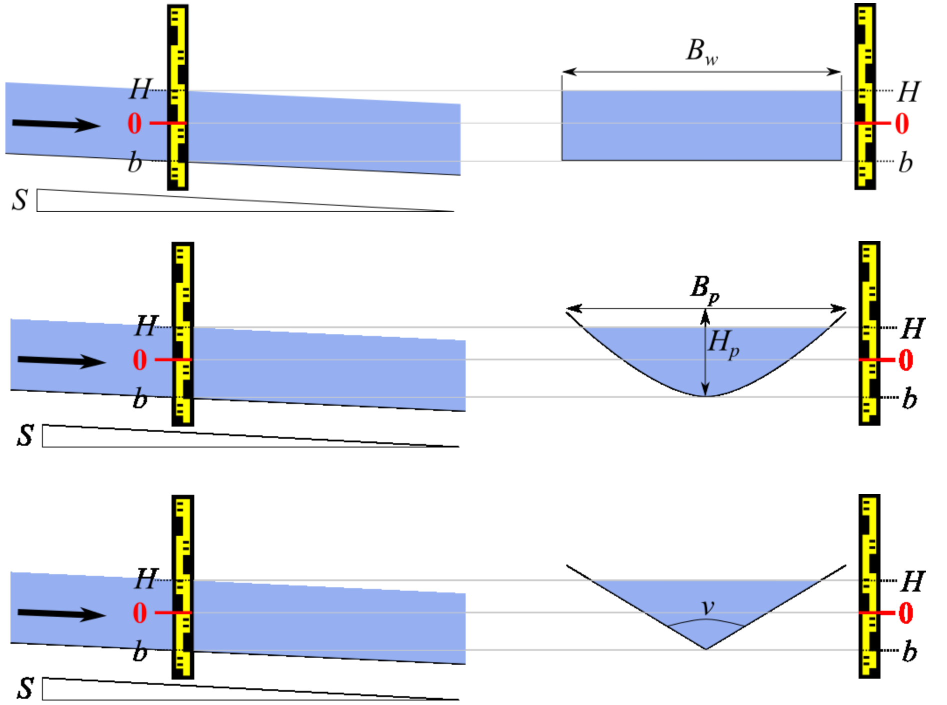
Discharge coefficient ≈ 0.31

Standard controls in BaRatinAGE

Section controls



Channel controls

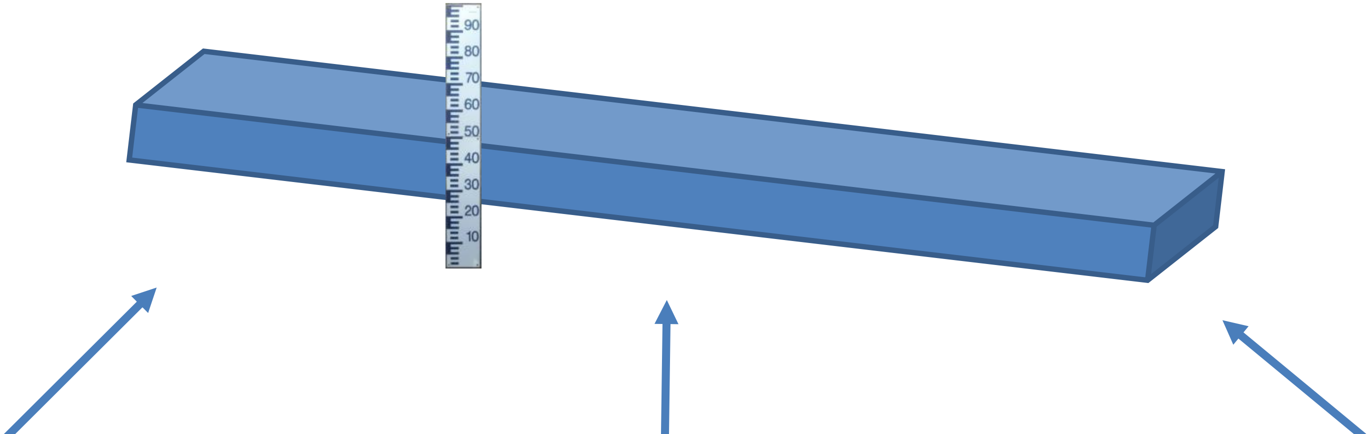


Unknown controls

$$Q = a(h-b)^c$$

Approximation of natural controls

- *Attention! The geometry of a channel control is an average of the section that extends downstream and upstream of the gauge*



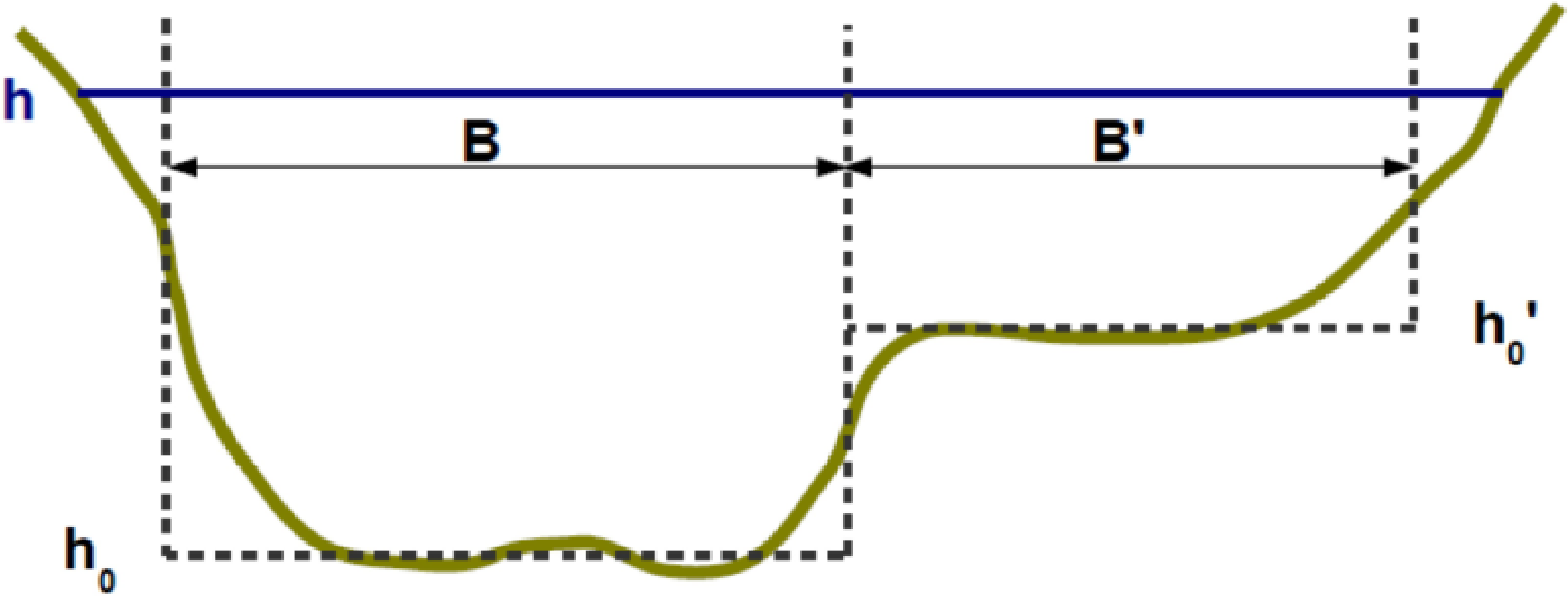
River Derwent, UK

Upper Truckee River, USA

Waimakariri, New Zealand

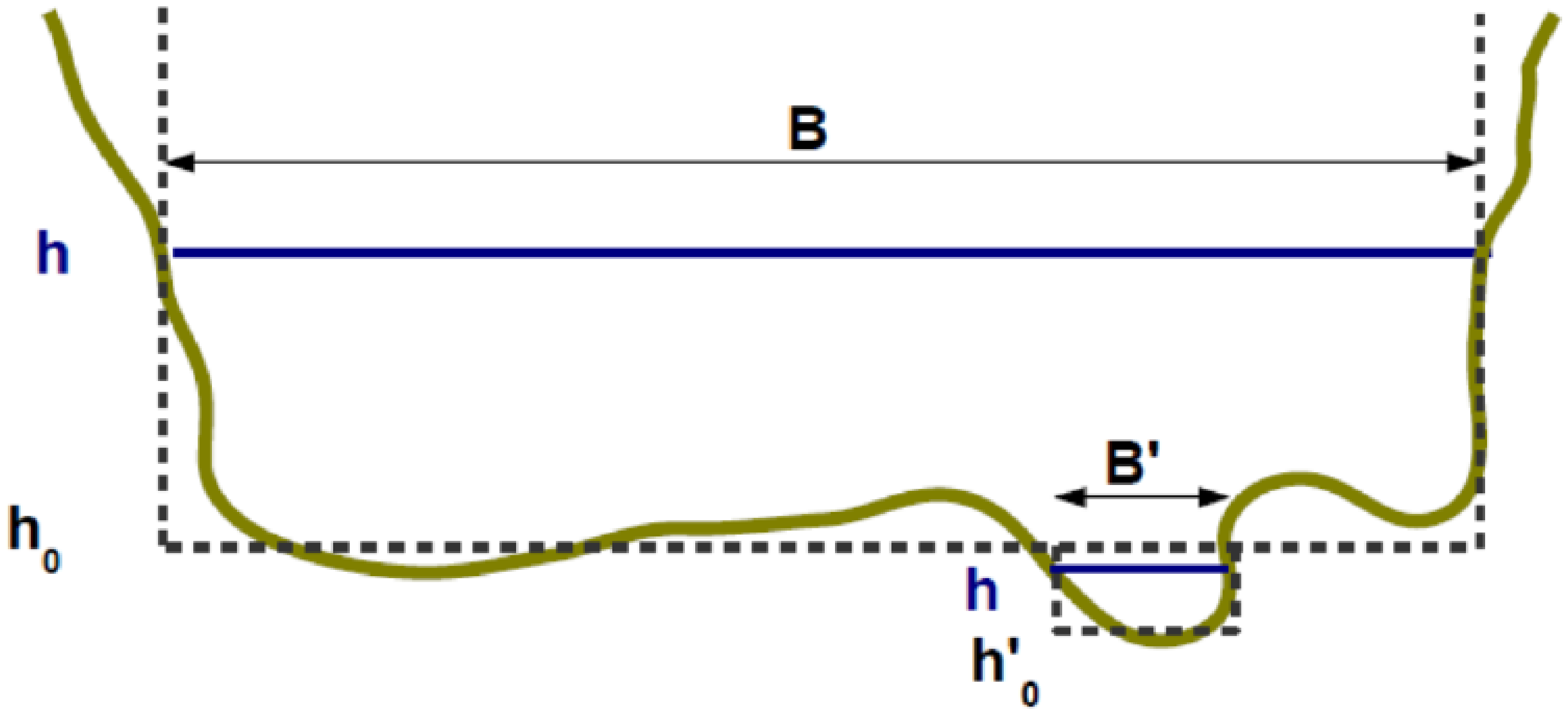
Approximation of natural controls

- Approximation of a compound channel using 2 recangular channels



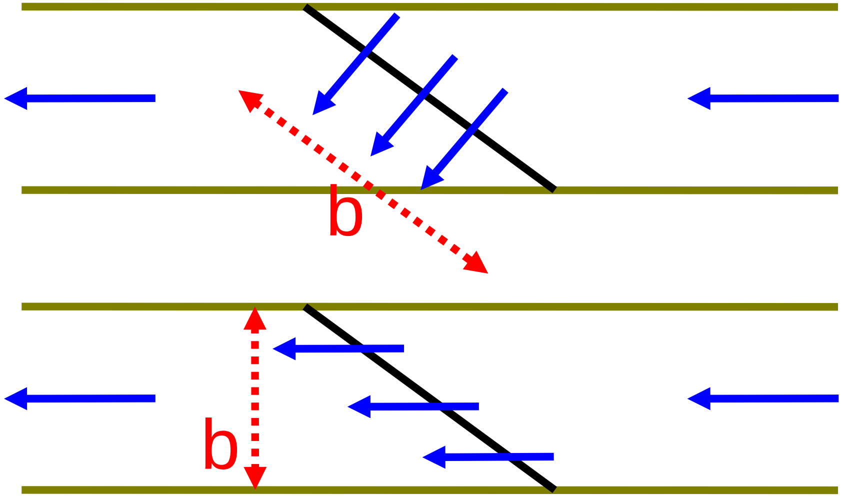
Approximation of natural controls

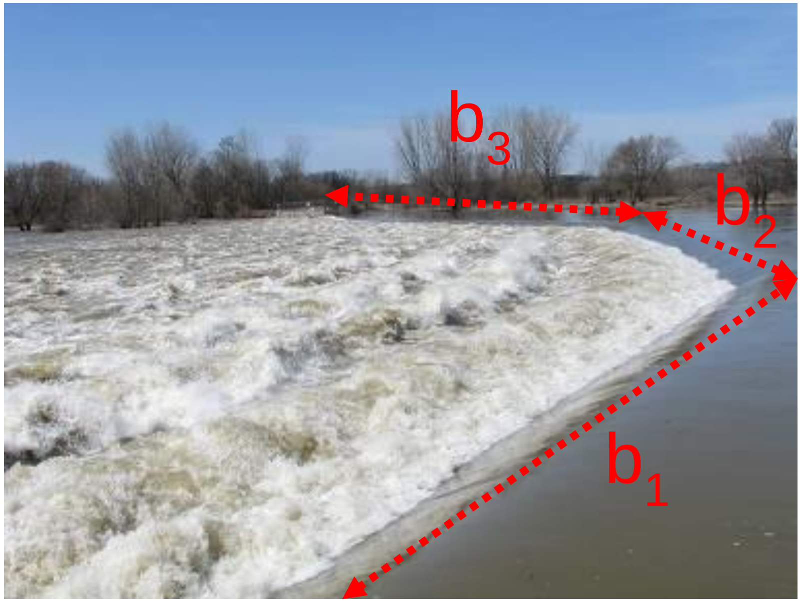
- Approximation of a complex critical section (natural riffle) by two nested rectangular weirs



Approximation of natural controls

- *Attention! The overflow width of the weir is counted perpendicular to the direction of flow*





Bayesian inference of hydraulic parameters

② Bayesian inference

Example of a weir:

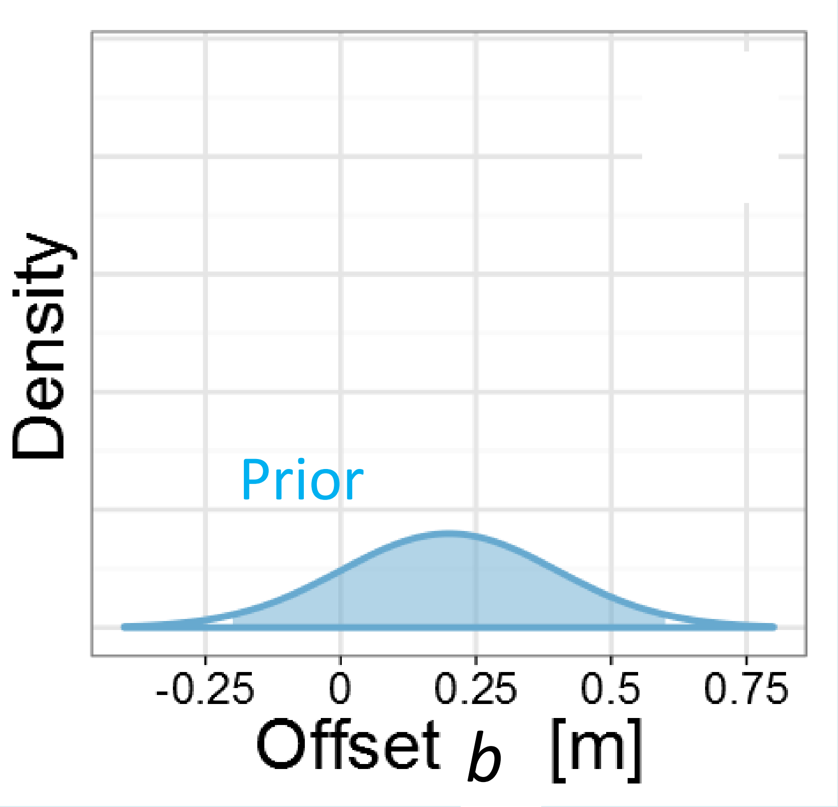
$$Q(h) = a(h - \boldsymbol{b})^{c}$$

The Altier River at La Goulette, France (EDF-DTG)

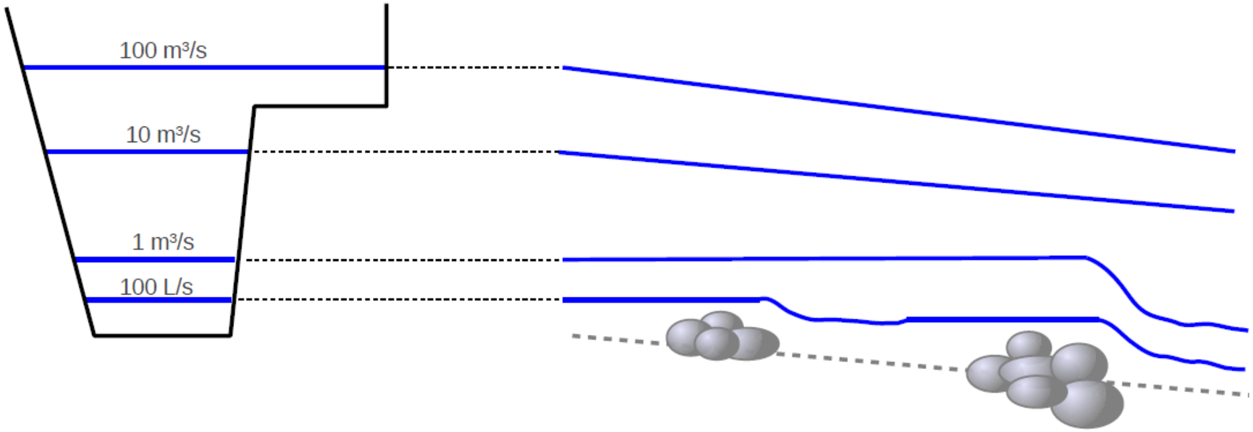
Bayesian inference of hydraulic parameters

② Bayesian inference

Prior knowledge: $b = 0.2\ \text{m} \pm 0.4\ \text{m}$



And now what?...



$$Q(h) = \begin{cases} 0 & \text{si } h < k_1 \\ a_1(h-b_1)^{c_1} & \text{si } k_1 \le h < k_2 \\ a_2(h-b_2)^{c_2} & \text{si } k_2 \le h < k_3 \\ a_3(h-b_3)^{c_3} & \text{si } k_3 \le h < k_4 \\ a_3(h-b_3)^{c_3} + a_4(h-b_4)^{c_4} & \text{si } k_4 \le h \end{cases}$$

To be estimated:
3 parameters
per control

Once we have the equation of the rating curve…
… we need to estimate parameters k_i, a_i, c_i (b_i are deduced by continuity)

The magics of Bayesian inference

The "posterior" distribution of the parameters of the rating curve can be computed using Bayes theorem:

Reverend
Thomas Bayes
(1702-1761)

Likelihood

Prior distribution

Posterior distribution

$$p(\theta|y) = \frac{p(y|\theta)\,p(\theta)}{p(y)}$$

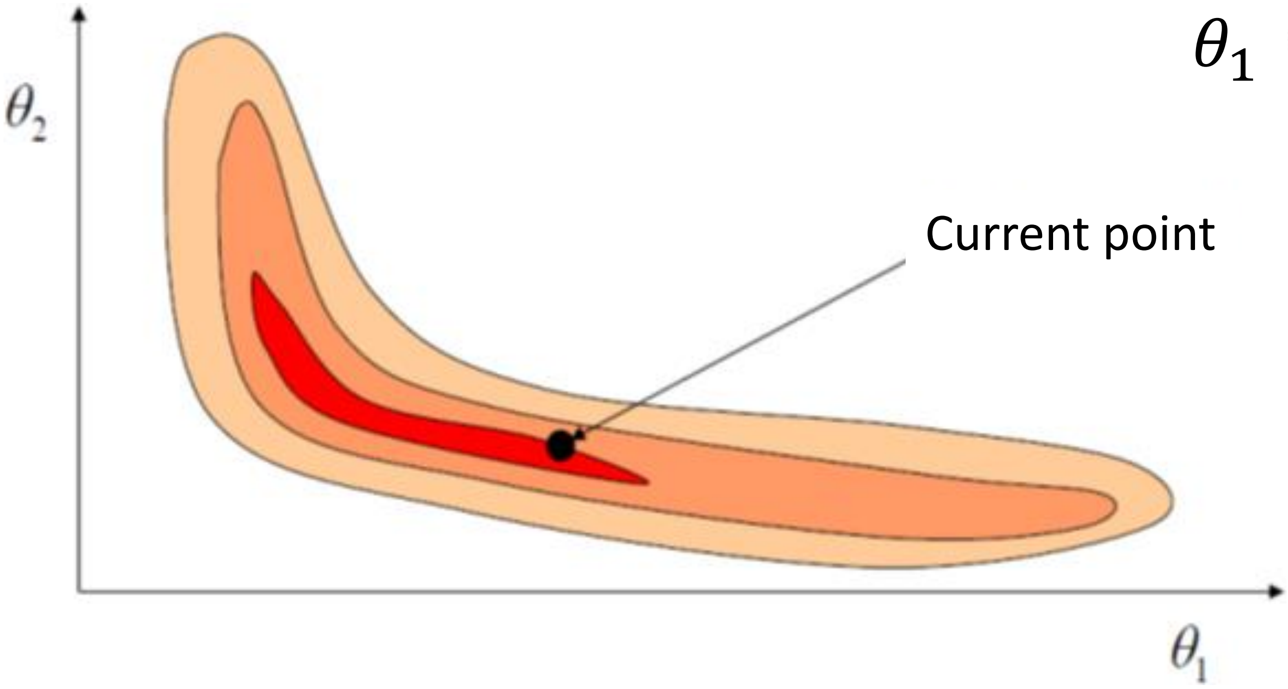
Normalization constant

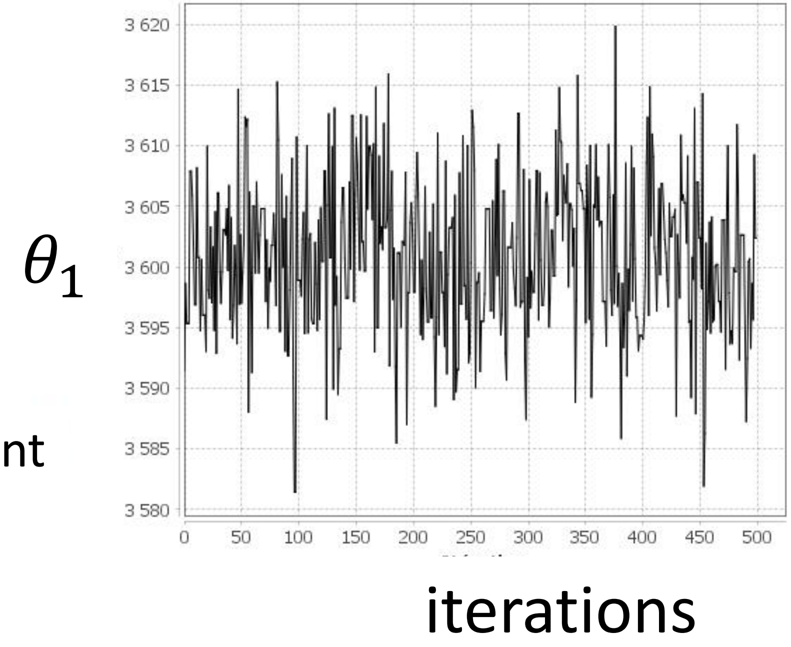
$\boldsymbol{y}$: observations (stage-discharge pairs: the gaugings)

$\boldsymbol{\theta}$: parameters of the rating curve

The magics of Bayesian inference

The posterior distribution is sampled by the Markov Chains Monte Carlo method (MCMC, Metropolis algorithm).





Bayesian inference of hydraulic parameters

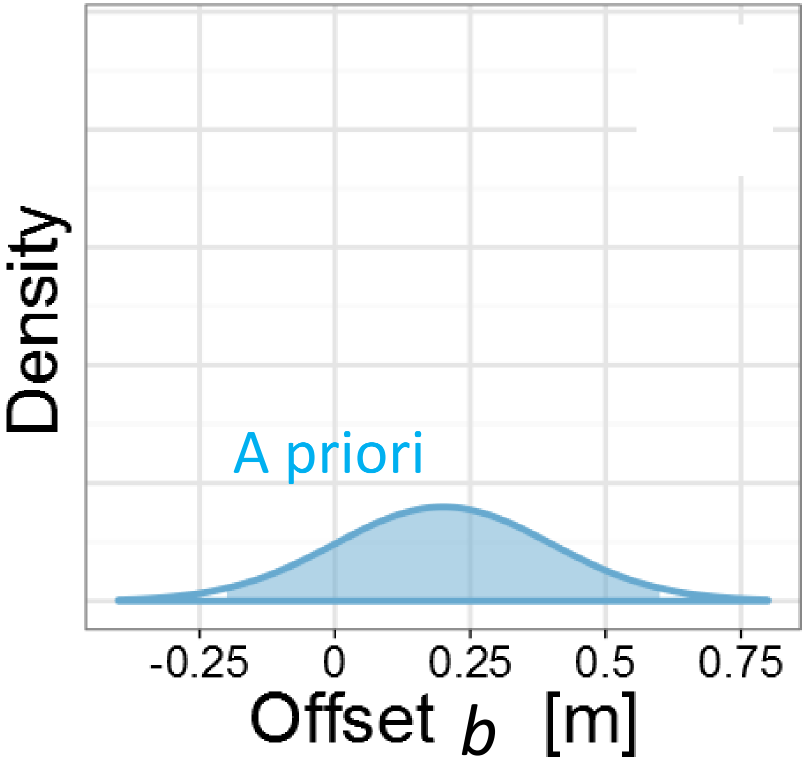
Example of a weir:

$$Q(h) = a(h - \boldsymbol{b})^c$$

Altier River at Goulette, France (EDF-DTG)

Bayesian inference of hydraulic parameters

Prior knowledge: b = 0.2 m ± 0.4 m

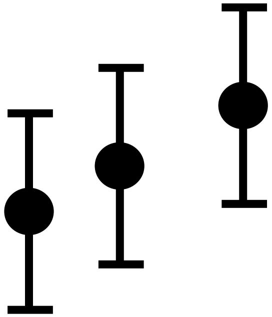


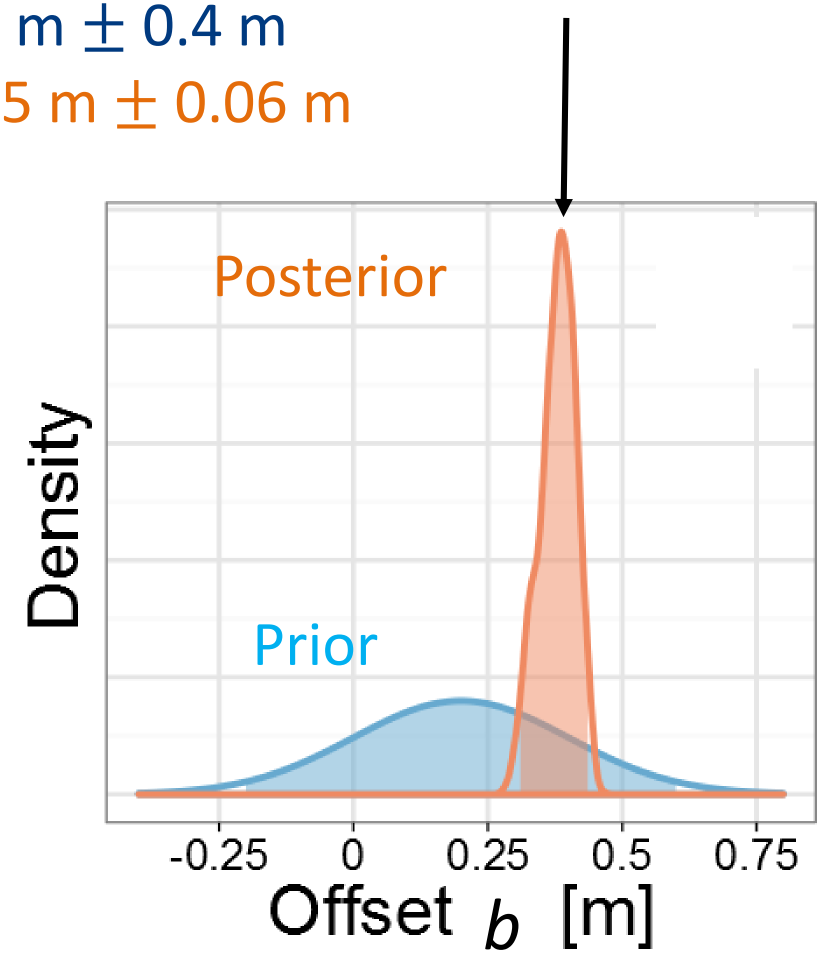
Bayesian inference of hydraulic parameters

Prior knowledge: b = 0.2 m ± 0.4 m

A posteriori : b = 0.35 m ± 0.06 m

Observations (gaugings):

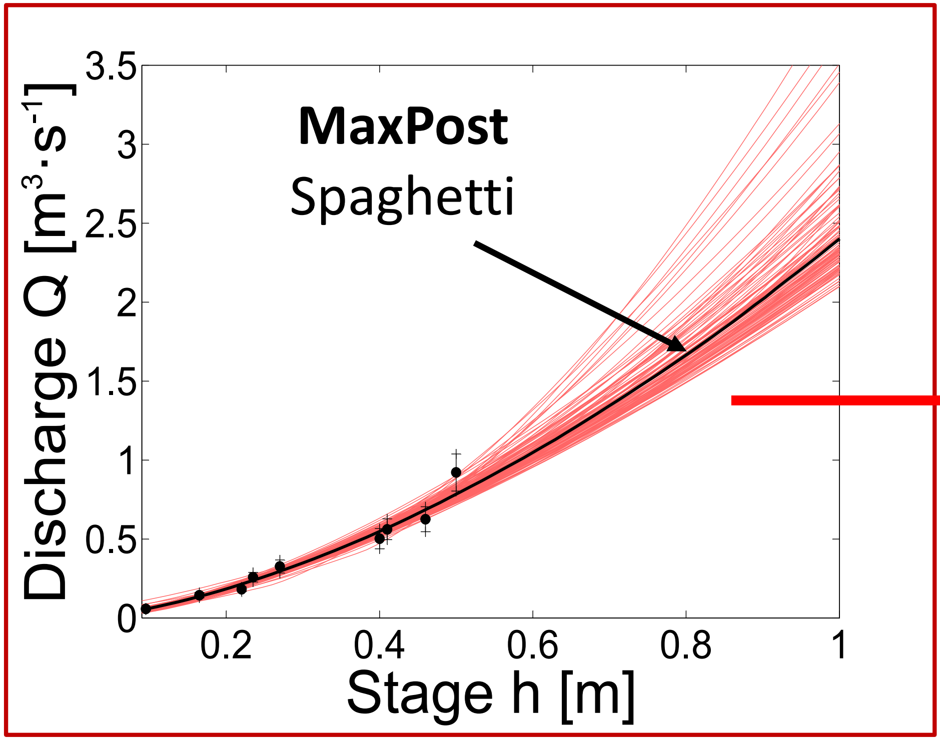


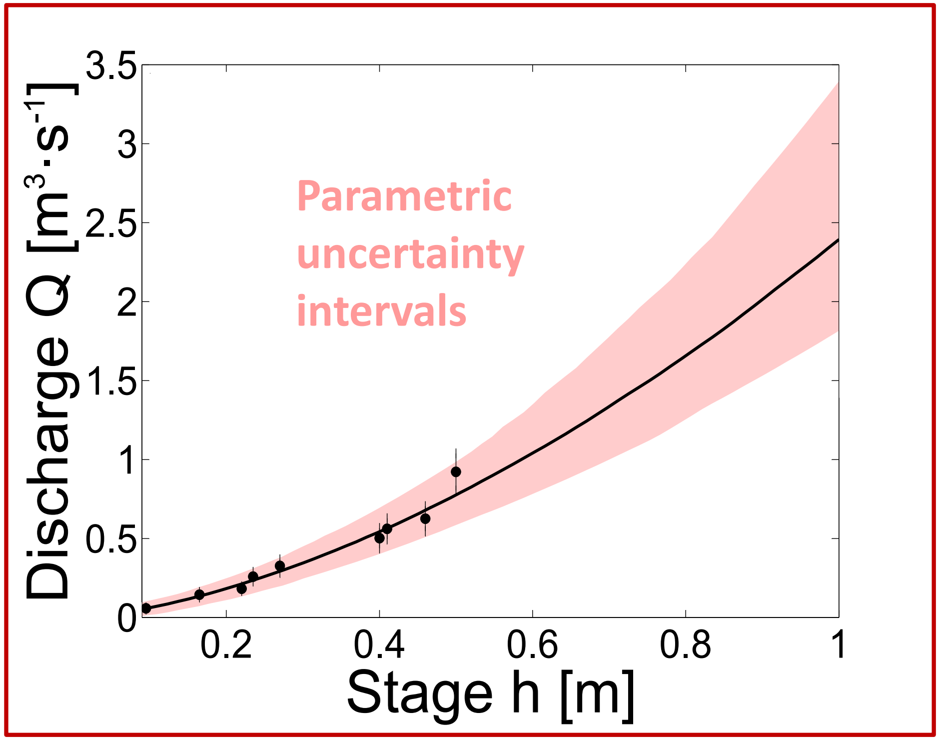


Realization
MaxPost (0.35 m)

The "spaghetti" approach

Posterior distribution is sampled using MCMC techniques





+ Structural/Remnant uncertainty:
What is lacking to explain the scatter of the gaugings around the rating curve

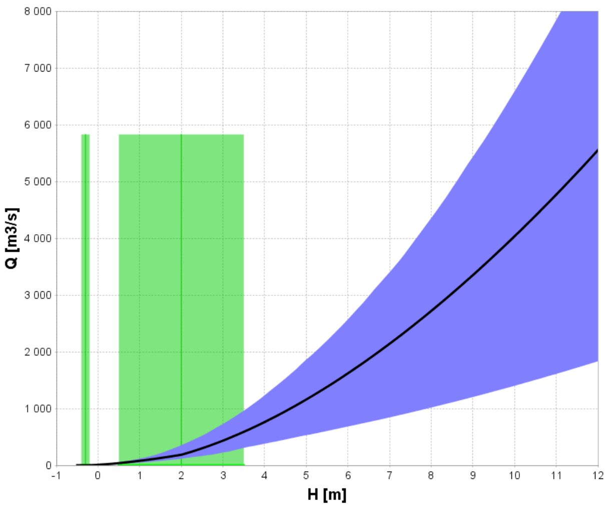
Bayesian estimation of the rating curve

The information contents of the measurements and of the hydraulic knowledge are combined.

Possible interpretations:

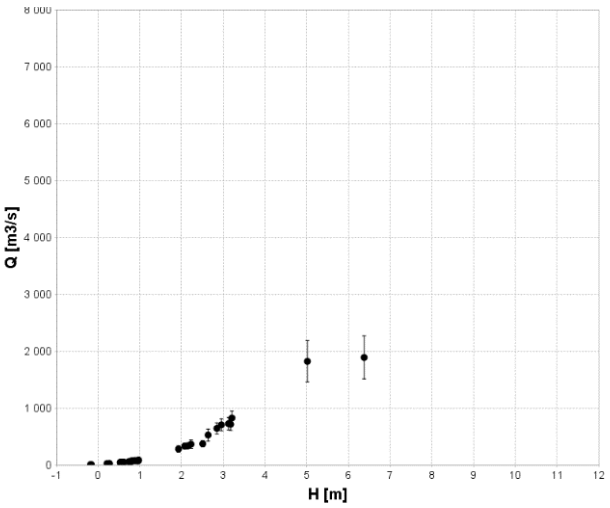
1. The hydraulic estimation is refined using the measurements
2. The rating is adjusted to the measurements under hydraulic constraints

Prior rating



*

Measurements (uncertain)



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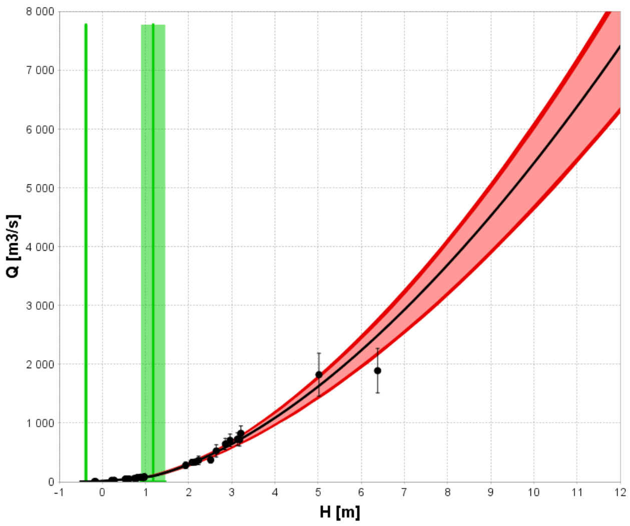
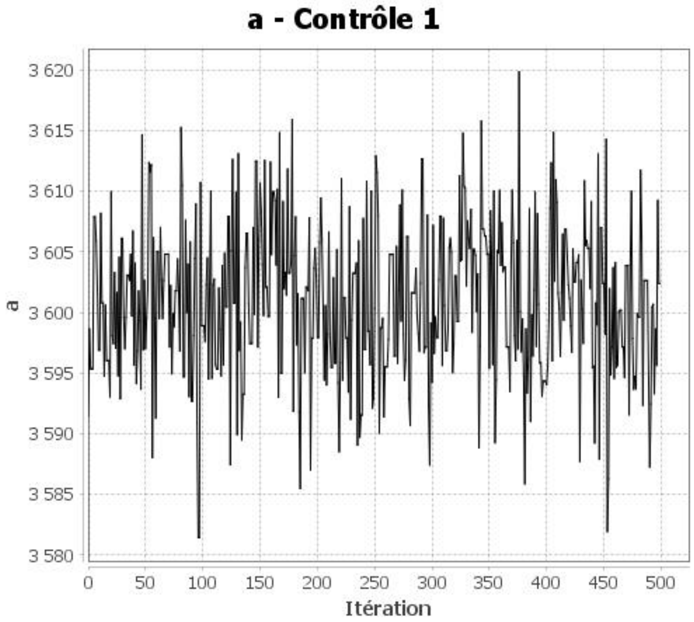
Posterior rating

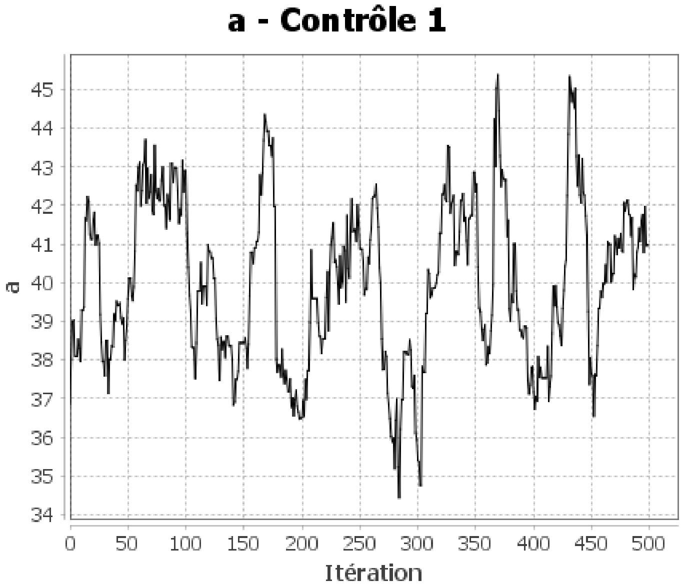


Example: the Ardèche at Sauze

Practically: Check MCMC realizations (traces)



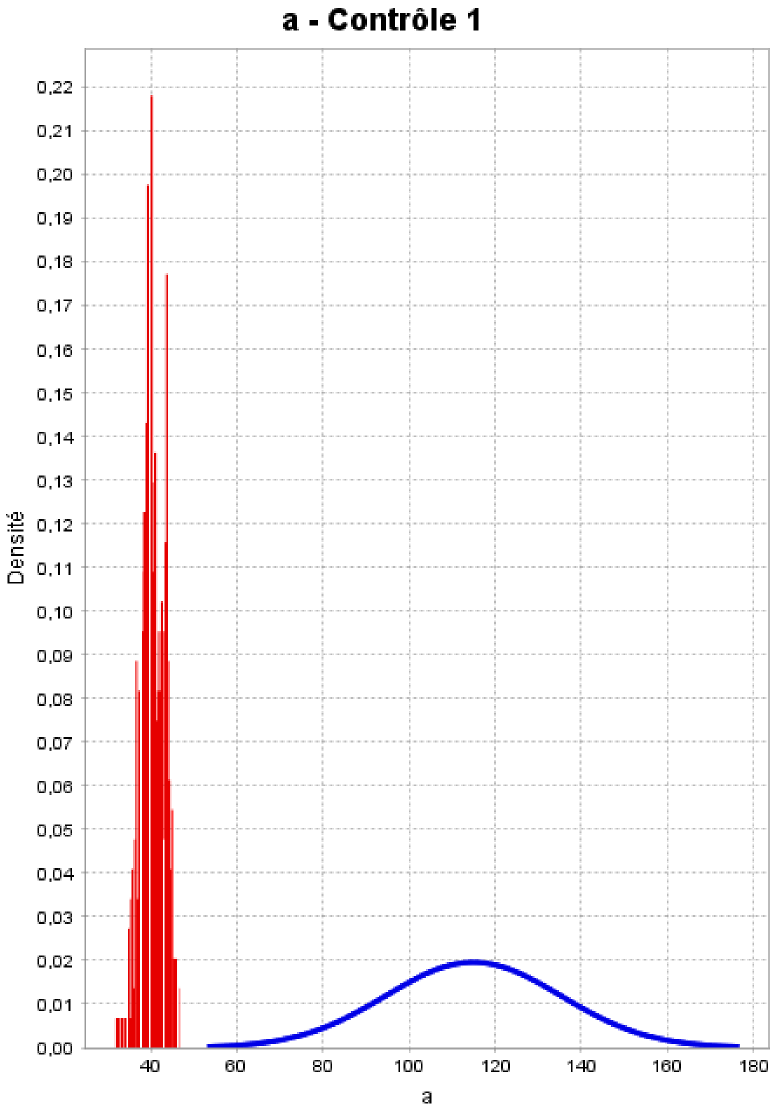
Good!
(‘stationary’ random walk)



Not so good…
(clear trends)

Practically:

Check there is no conflit between prior and posterior



Not good!

Practically:

Check there is no conflit between prior and posterior

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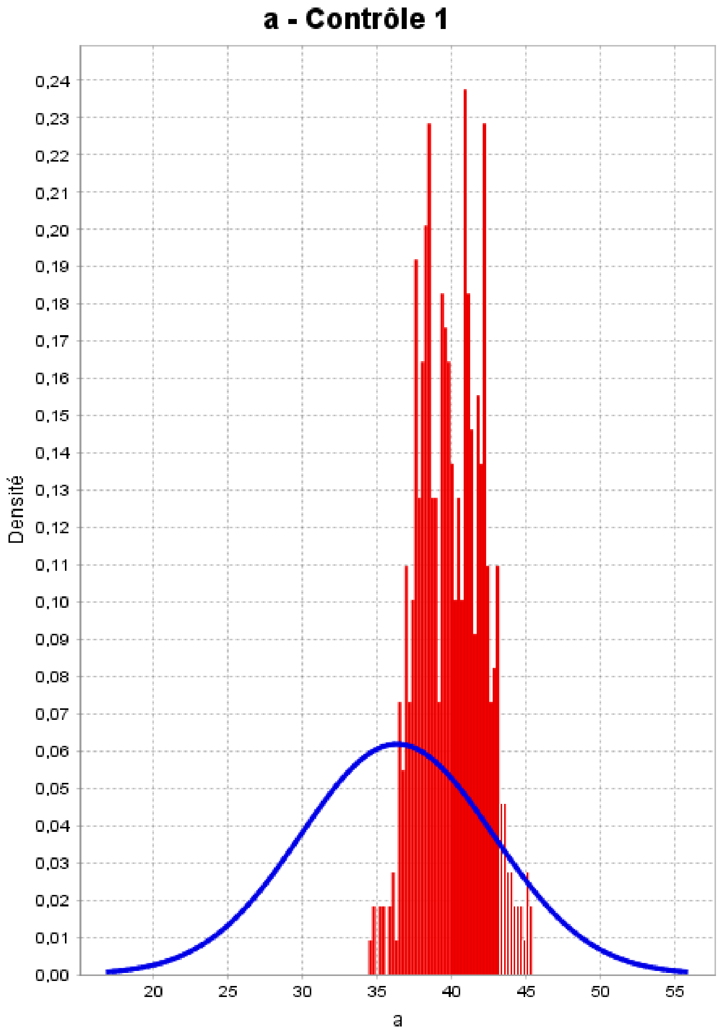
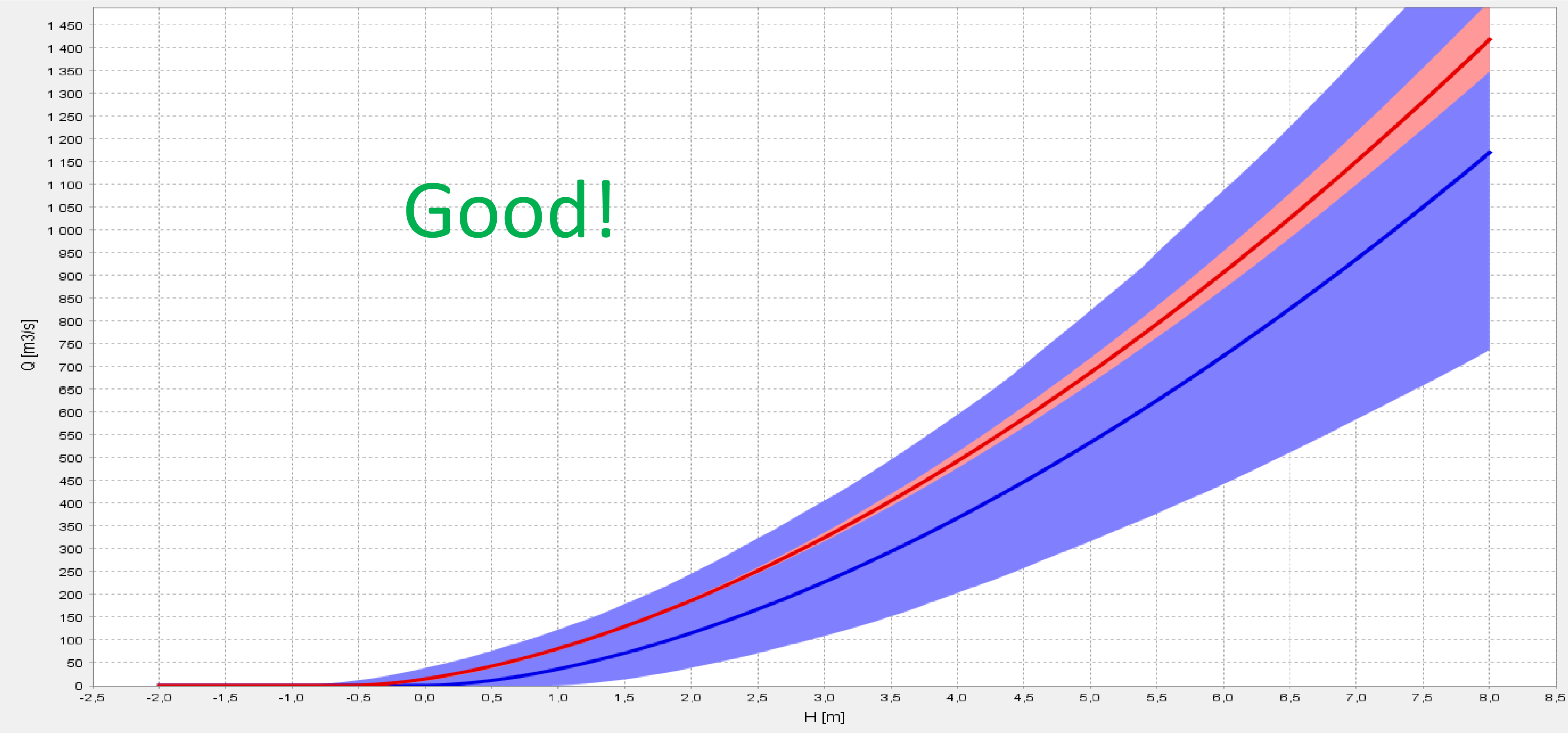
Check there is no conflit between prior and posterior



Good!

Practically:

Check there is no conflit between prior and posterior


Good!

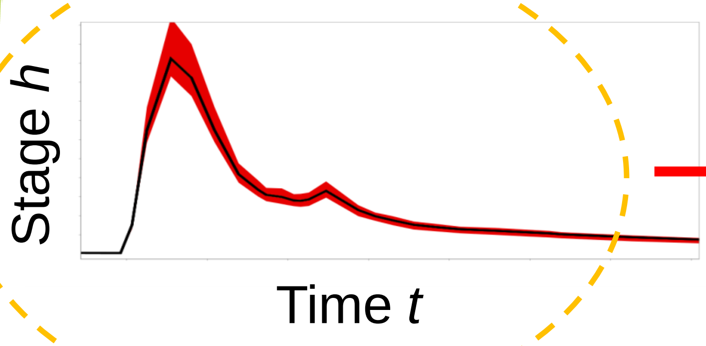
Practically:

Check there is no conflit between prior and posterior

If there appears to be a conflict:

- ✓ Check that the calculations went well (convergence of MCMC iterations)
- ✓ Check the values of the priors (do not set them using the results or the gaugings used in the estimation!)
- ✓ Review your assumptions on hydraulic controls, test other hydraulic configurations
- ✓ Check the gaugings and their uncertainties

Establishing probabilistic streamflow data



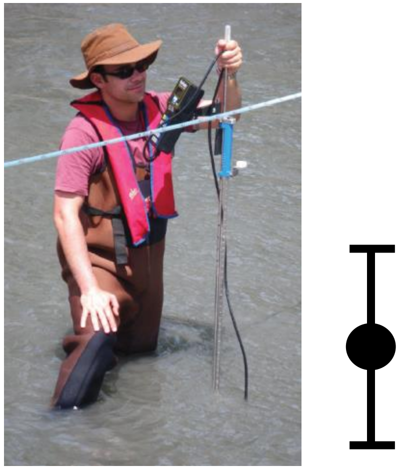
Water level series

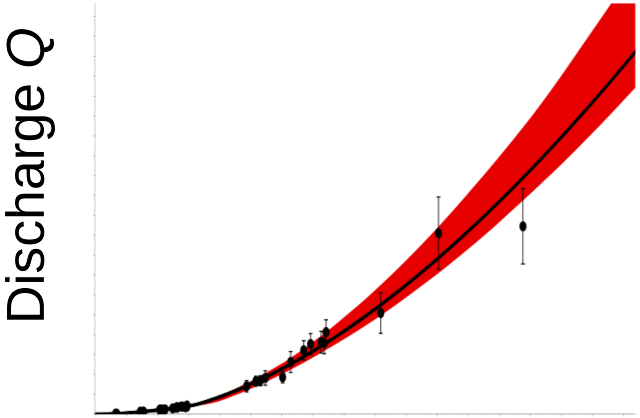
h(*t*)

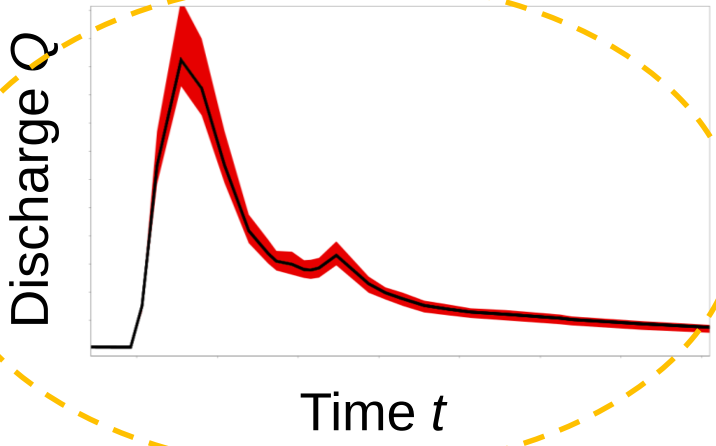
Gaugings
(Q_i,h_i)

Rating curve
Q(h)

Streamflow series
Q(*t*)



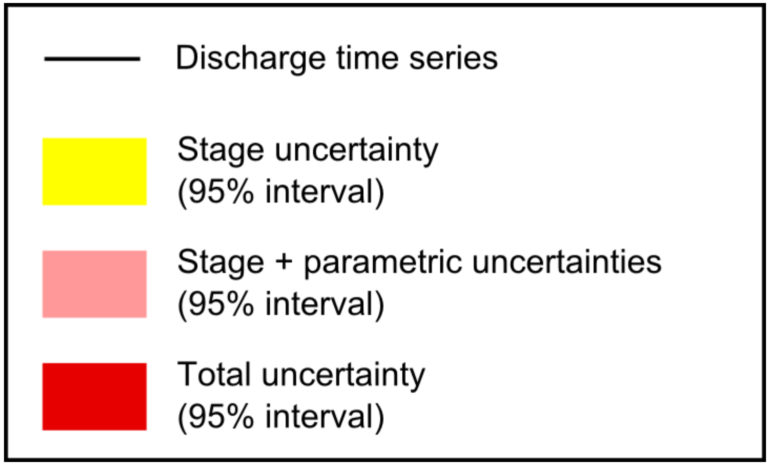




Uncertainty budgets

Uncertainty budgets help to rank sources of error and improve the measurement process

- ✓ Reflects the measurement uncertainties of the stage records
- ✓ Reflects the (limited) information contents of the priors and observations (the gaugings)
- ✓ Combines parametric and structural uncertainties
- ✓ Structural uncertainty reflects the limitations of the rating curve model for describing the real hydraulic conditions of the site (complex controls, shifts, hysteresis, variable backwater…)



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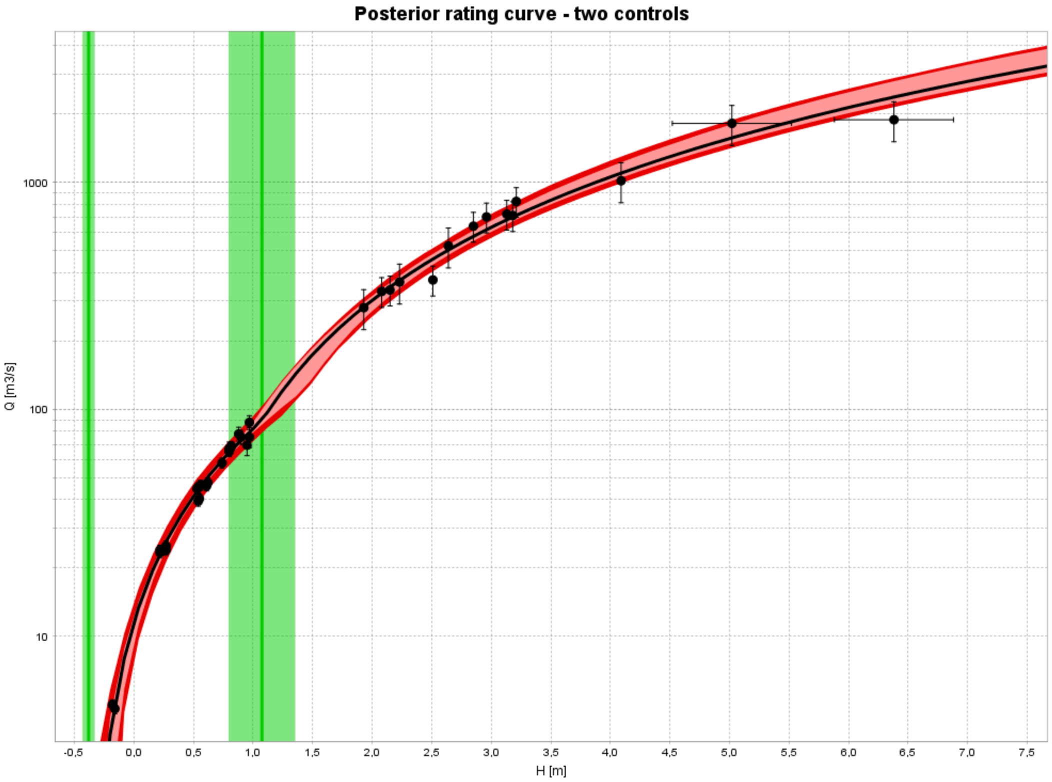
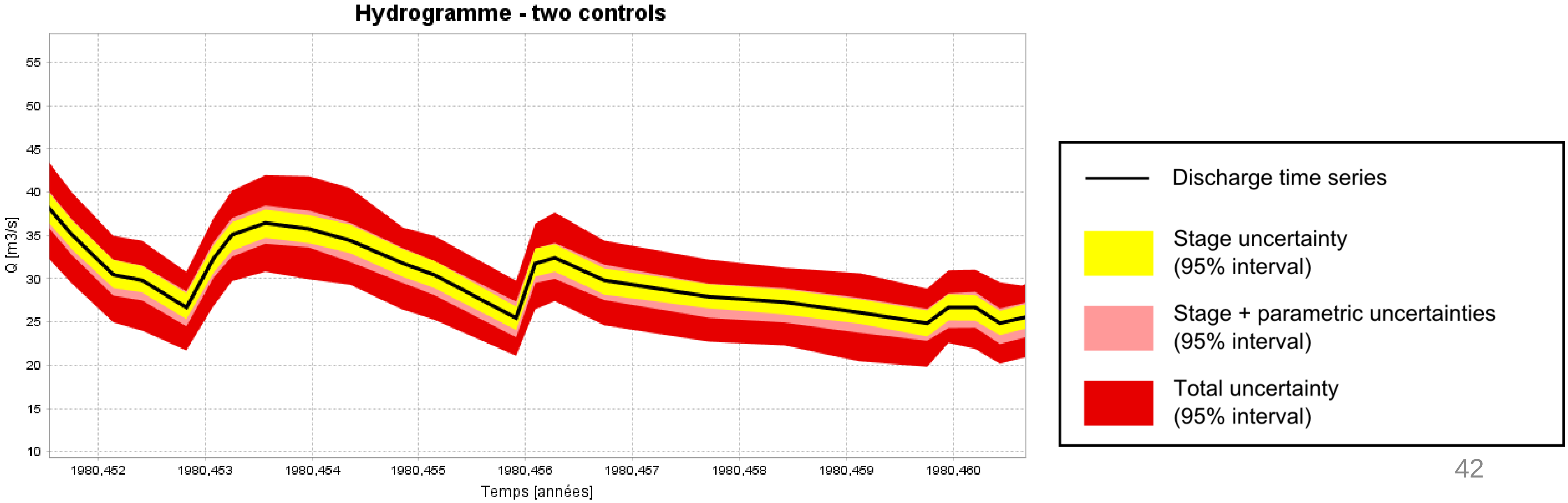


Posterior rating curve - two controls

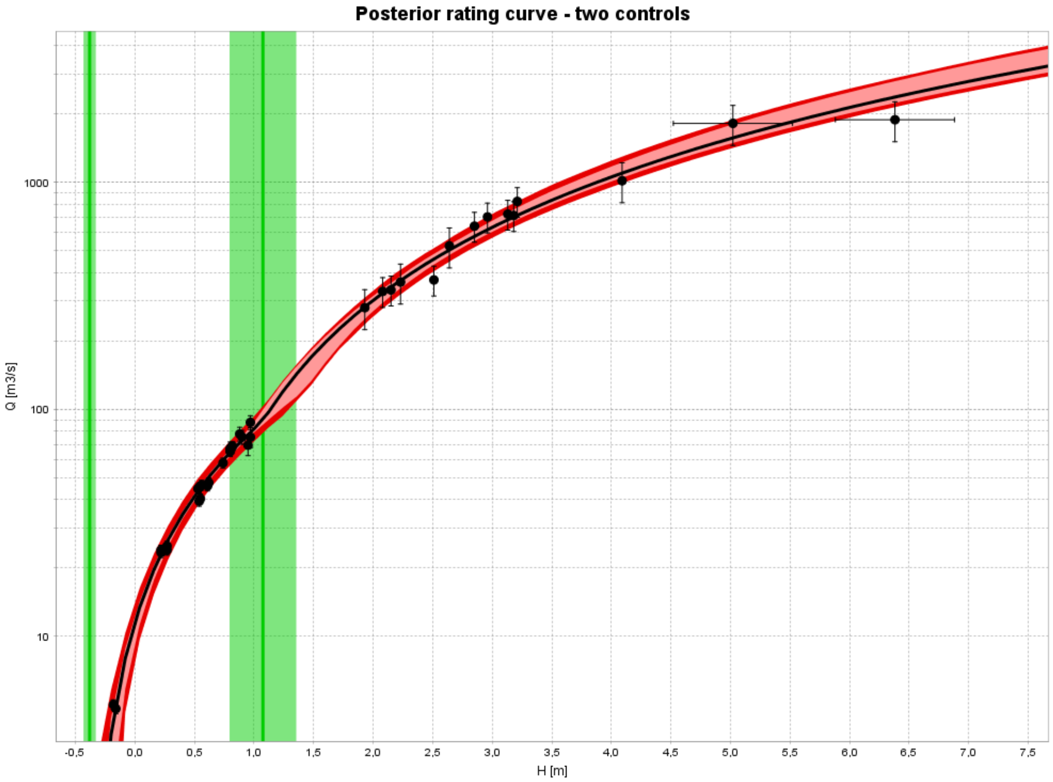
Low flows



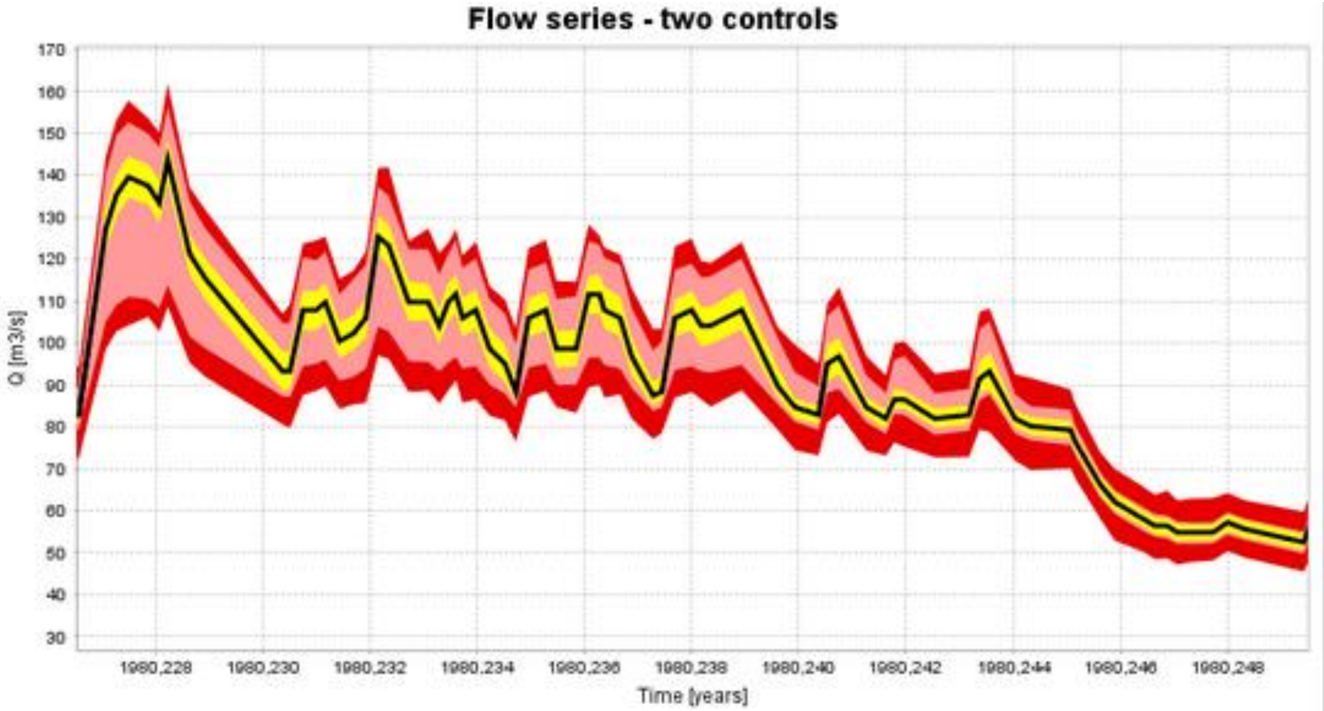
Hydrogramme - two controls

Uncertainty budgets

Uncertainty budgets help to rank sources of error and improve the measurement process



Intermediate flows



— Discharge time series

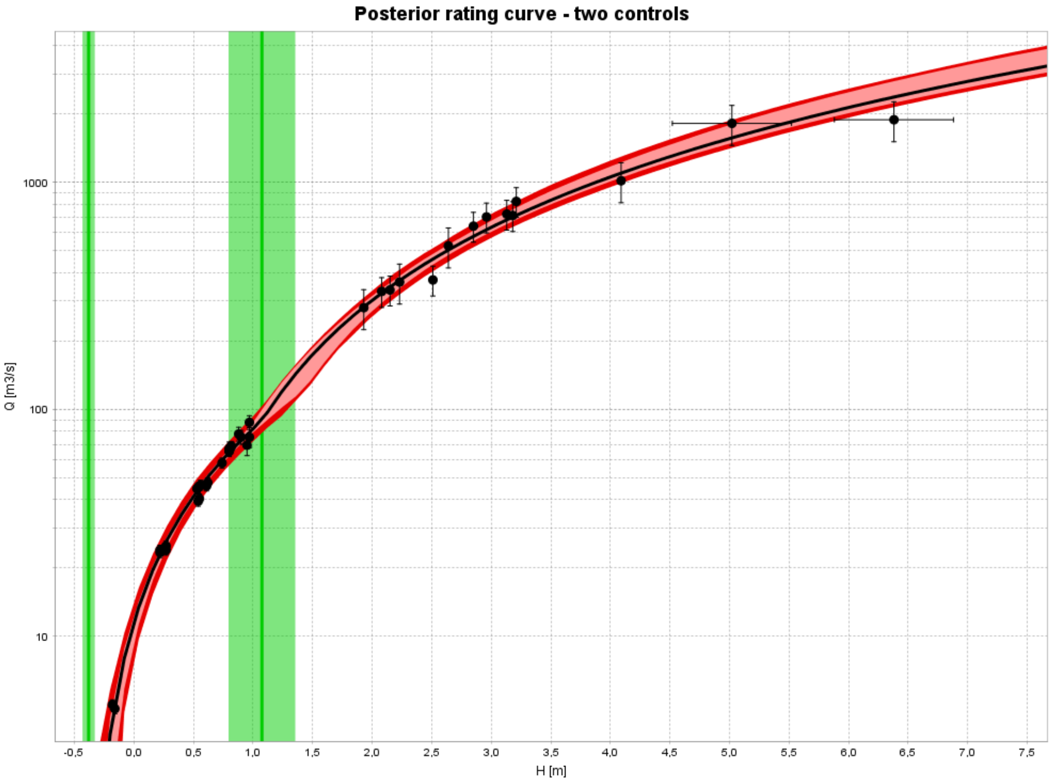
🟨 Stage uncertainty (95% interval)

🟥 Stage + parametric uncertainties (95% interval)

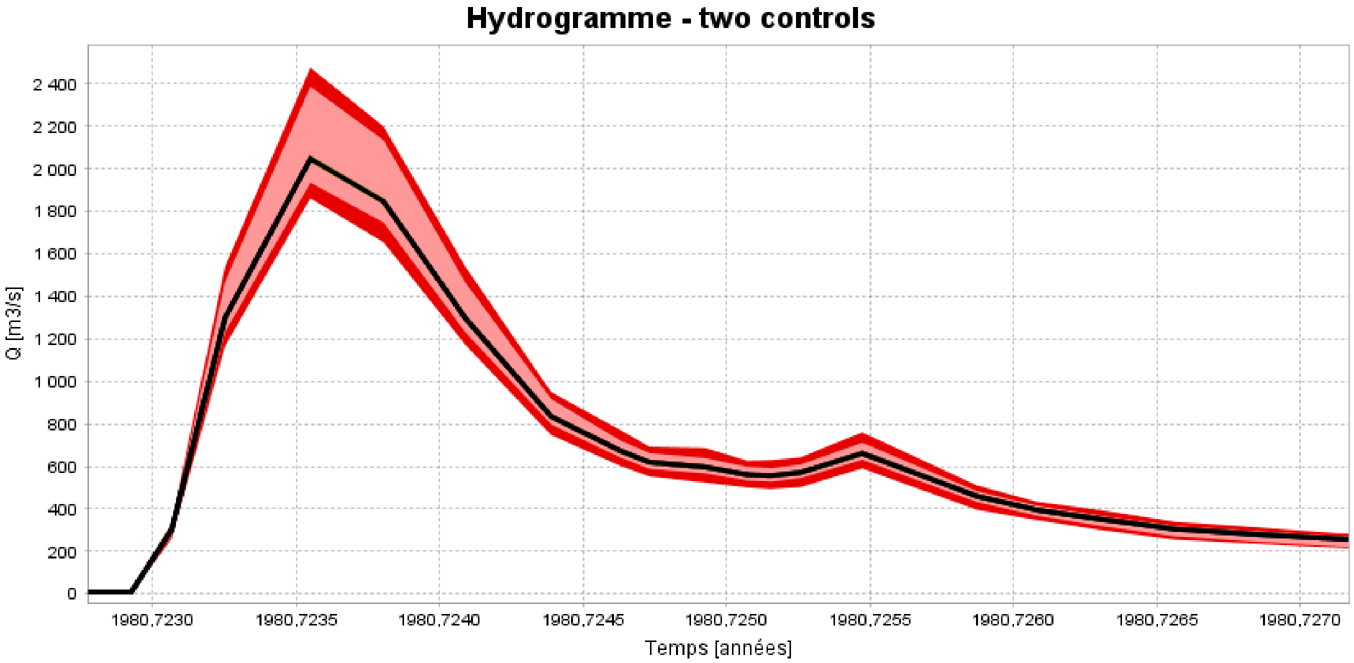
🟥 Total uncertainty (95% interval)

Uncertainty budgets

Uncertainty budgets help to rank sources of error and improve the measurement process


Posterior rating curve - two controls

High flows


Hydrogramme - two controls

— Discharge time series

🟨 Stage uncertainty (95% interval)

🟥 Stage + parametric uncertainties (95% interval)

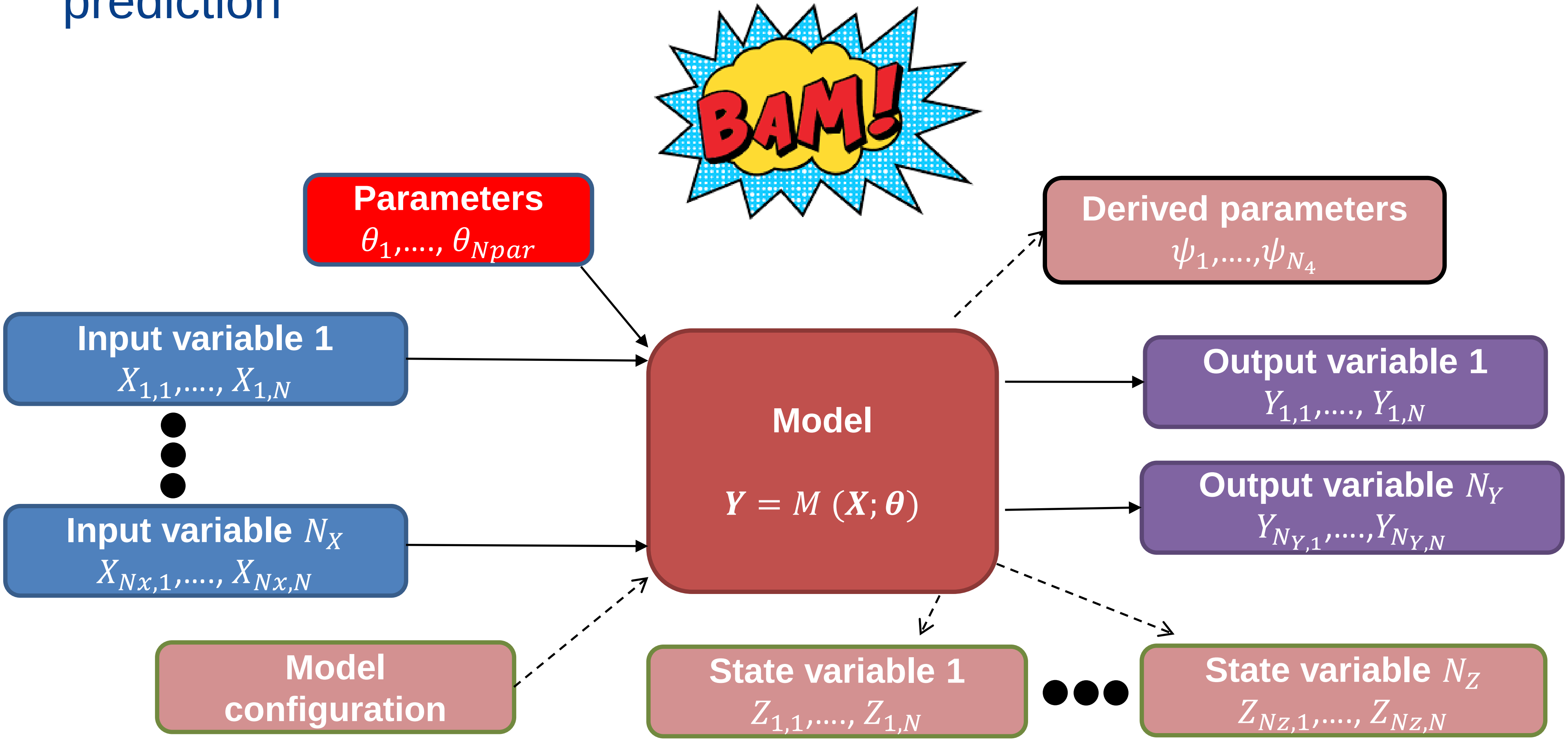
🟥 Total uncertainty (95% interval)

The BaRatin method for rating curves

- ✓ Introduction
- ✓ Hydraulic principles behind the rating curve
- ✓ Rating curve estimation
- ✓ Going further: more on BaRatin

Extension of BaRatin: *BaM!*

Calculation code for estimating any model and using it for prediction



Parameters
$\theta_1,\ldots,\theta_{Npar}$

Derived parameters
$\psi_1,\ldots,\psi_{N_4}$

Input variable 1
$X_{1,1},\ldots,X_{1,N}$

Input variable N_X
$X_{Nx,1},\ldots,X_{Nx,N}$

Model

$Y = M\,(X;\theta)$

Output variable 1
$Y_{1,1},\ldots,Y_{1,N}$

Output variable N_Y
$Y_{N_{Y,1}},\ldots,Y_{N_{Y,N}}$

Model configuration

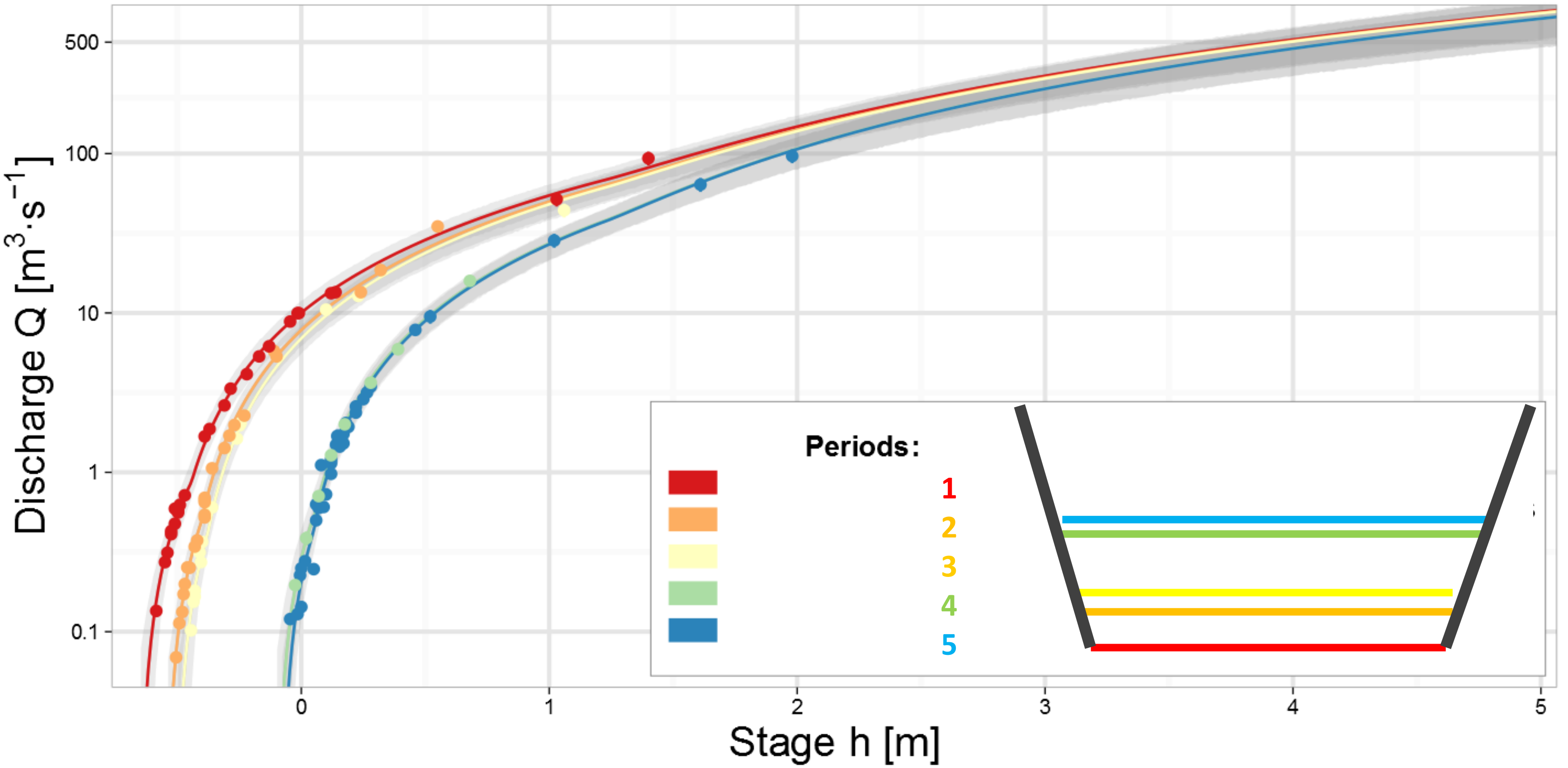
State variable 1
$Z_{1,1},\ldots,Z_{1,N}$

State variable N_Z
$Z_{Nz,1},\ldots,Z_{Nz,N}$

Schematisation of *BaM!* (Renard, 2017)

Rating shifts due to bed evolution

BaRatin-SPD (stage-period-discharge): rating changes due to bed evolution at known times



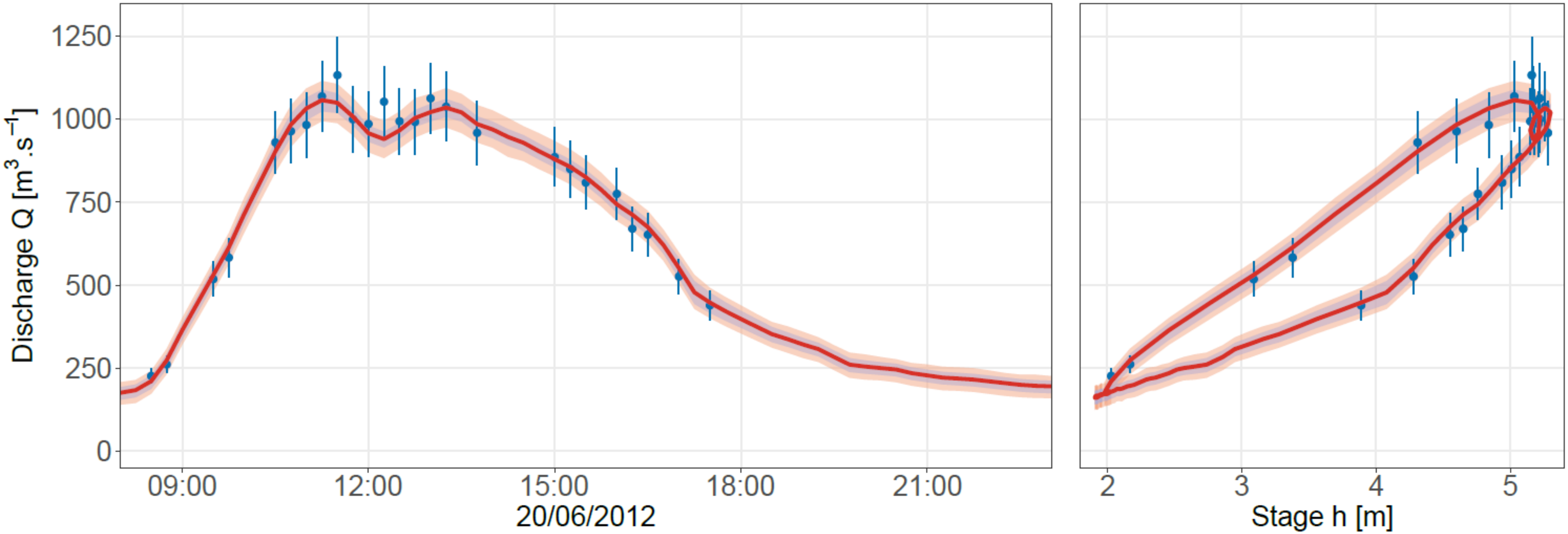
PhD of Valentin Mansanarez (2016)

Mansanarez et al. (2019)

$$Q\left(h, i_{\text{period}}\right)$$

Complex rating curves

BaRatin-SGD (stage-gradient-discharge): hysteresis due to transient flow

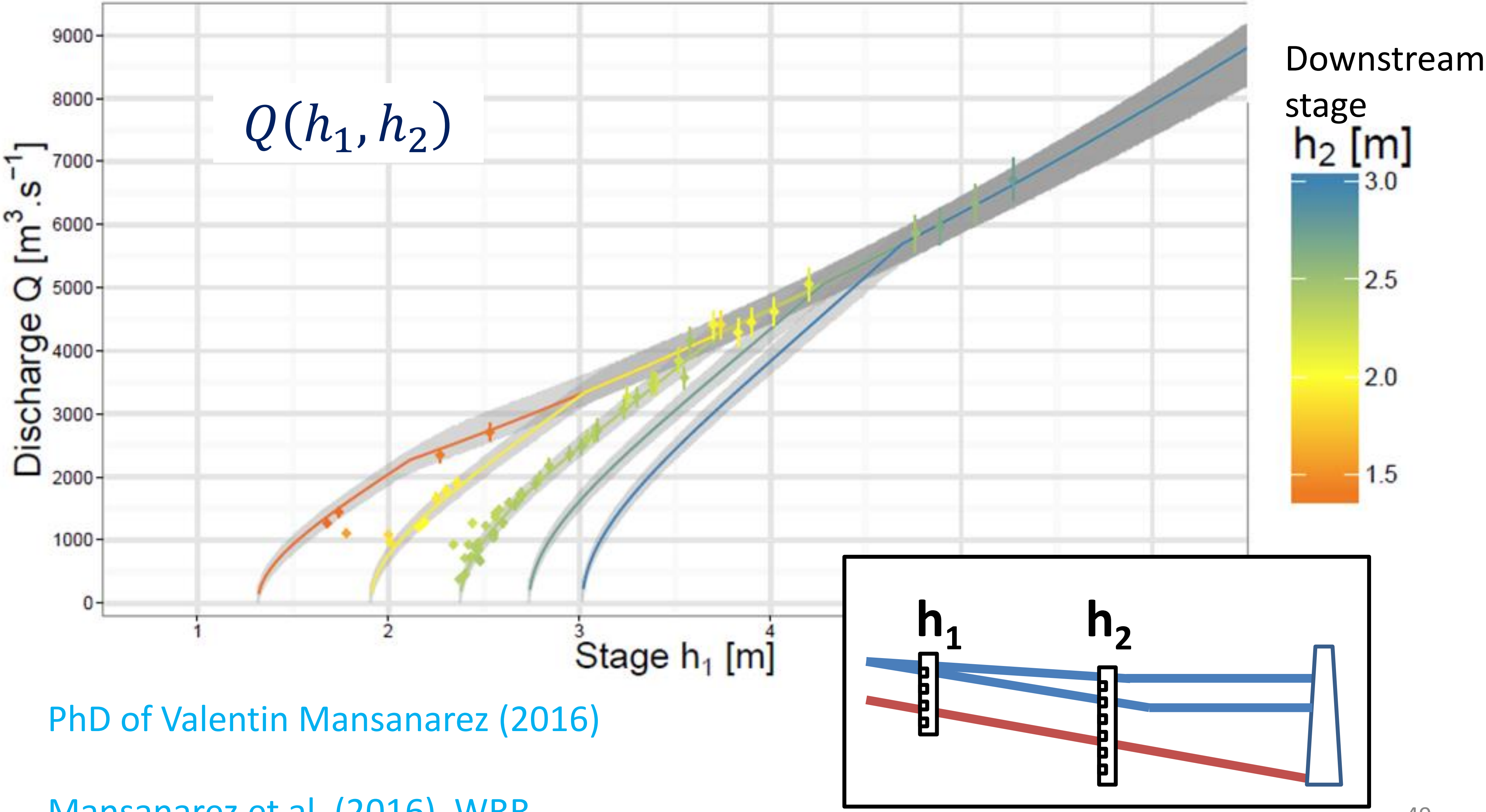


PhD of Valentin Mansanarez (2016)

$$Q\left(h,\frac{\partial h}{\partial t}\right)$$

Complex rating curves

BaRatin-SFD (stage-fall-discharge): variable backwater



PhD of Valentin Mansanarez (2016)

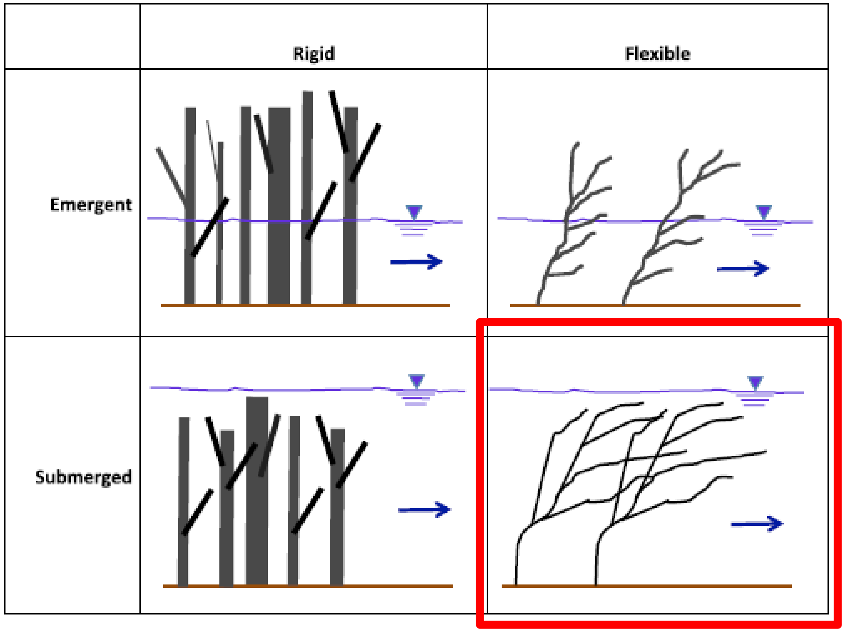
Mansanarez et al. (2016), WRR

Rating curves affected by aquatic vegetation

Modified channel control equation with vegetation roughness added

$$Q(h,t) = \frac{1}{\sqrt{n_b^2 + n_v^2}} B\sqrt{S_0}(h(t) - b)^c$$

Perret et al. (2021)



Shields Jr. (2017)